A DYNAMIC GAME APPROACH TO THE WORLD ORANGE JUICE MARKET AND INVENTORY PLANNING

BY

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To the memory of my father.

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All praise is due to God alone, the Sustainer of all the worlds, the Most Gracious, the Dispenser of Grace.

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TABLE OF CONTENTS

page

ACKNOWLEDGMENTS iii

LIST OF TABLES viii

LIST OF FIGURES ix

KEY TO SYMBOLS xi

ABSTRACT xiii

CHAPTERS

1. INTRODUCTION 1

    Background 1
    Current State of the Orange Juice Market 5
    Problem Statement 10
    Objectives 16
    Methodology, Scope, and Organization of the Presentation 16

2. REVIEW OF LITERATURE AND THEORETICAL FOUNDATIONS 19

    Dynamic Optimization and Production-Inventory Theory 19
    Industrial Organization/Economics and the Theory of Duopoly . . 29
        Quantity and Price Competition Models 30
        Conjectural Variation (CV) Models 39
        Collusion Models 41
    Dynamic Game Theory 45
    Previous Studies on Domestic Orange Juice Industry 60

3. THEORETICAL MODELS OF THE WORLD FCOJ MARKET 64

4. CHARACTERIZATION OF THE OPTIMAL SOLUTIONS: A THEORETICAL ANALYSIS 78

The Dynamic Duopoly Quantity Game (DDQG) Model 79
    Nonspatial Open-loop Solutions to the DDQG Model . . . 79
    Nonspatial Feedback Solutions to the DDQG Model 84
    Nash and Stackelberg Strategies in the DDQG Model . . . 85
    General Conjectural Variations (CV) for the DDQG Model 88
    Tacit/Implicit Price Collusion 89
        Nash strategies 89
        Stackelberg strategies 90
    Spatial Solutions to the DDQG Model 90
    The Special Case of a Linear Demand Function 91
The Dynamic Duopoly Price Game (DDPG) Model 95
    Nonspatial Solutions to the DDPG Model 96
    Spatial Solutions to the DDPG Model 98
    The Special Case of a Linear Demand Function 99
Equivalent Strategies in DDQG and DDPG Models 102

5. EMPIRICAL RESULTS AND DISCUSSION 103

Specification of the Empirical Models 103
    The DDQG Model 107
    The DDPG Model 109
Data 111
Numerical Results 113
    The DDQG Model 113
    The DDPG Model 117

6. SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE RESEARCH 123

Summary 123
Conclusions 125
Limitations and Suggestions for Future Research 126

APPENDICES

A A STATIC ANALYSIS OF DUOPOLY GAMES 130

B A DYNAMIC ANALYSIS OF AN INVENTORY PROBLEM . 139

C RELATIONSHIP POSSIBILITIES BETWEEN TWO DIFFERENT GAMES 141

D DATA 143

REFERENCES 157

BIOGRAPHICAL SKETCH 169

LIST OF TABLES

Table page

1-1 Utilization of Florida round oranges (in million bozes) 6

1-2 The payoff matrix for Florida and Brazil 52

2-2 The payoff matrix in stage two 53

2-3 The payoff matrix for the two-stage game 54

5-1 Estimates of the inverse demand functions for the DDQG model 114

5-2 Estimates of the quantity reaction functions for the DDQG model 116

5-3 Estimates of the quantity conjectural variations, time and seasonal factors in the DDQG model 117

5-4 Estimates of the demand functions for the DDPG model 118

5-5 Estimates of the price reaction functions for the DDPG model 119

5-6 Estimates of the price conjectural variations and initial inventory in the DDPG model 120

D-1 Florida FCOJ data (thousand SSE gallons) July 1990 - June 1993 143

D-2 Brazilian FCOJ data (thousand SSE gallons) July 1990 - June 1993 . . . 144

D-3 Florida FCOJ price (deflated) and marginal cost data (US $ per thousand SSE gallons) July 1990 - June 1993 145

D-4 Brazilian FCOJ price (deflated) and marginal cost data (US $ per thousand SSE gallons) July 1990 - June 1993 146

D-5 Data of exogenous variables (July 1990 - June 1993) 147

LIST OF FIGURES

Figure page

1-1 Florida and Brazilian orange juice production, US consumption, and imports 1979/80 - 1993/94 2

1-2 Florida FCOJ situation in 1992-93 season 8

1-3 The world orange juice situation in 1990-91 9

1-4 Brazilian FCOJ exports by destination in 1992-93 11

1-5 Brazilian FCOJ situation 1979-1992 12

1-6 Bulk FCOJ FOB prices in the U. S. 14

2-1 Marginal cost of storage 24

2-2 A Cournot duopoly 34

2-3 A Bertrand duopoly 36

2-4 Edgeworth duopoly model 37

2-5 A price-leadership duopoly 44

2-6 A duopoly game with Nash and Stackelberg solutions 51

2-7 A duopoly game without Nash but with Stackelberg solutions 51

3-1 The schematic of the world FCOJ market under study 69

D-1 Florida FCOJ movement 149

D-2 Brazilian FCOJ movement 150

D-3 FCOJ movement in the US market 151

D-4 FCOJ movement in the European market 152

D-5 FCOJ price series (deflated) in the US 153

D-6 FCOJ price series (deflated) in Europe 154

D-7 Brazilian FCOJ monthly production 156

KEY TO SYMBOLS

$u_{ik}(t)$: quantity sold from player i in market k at time t.

$q_i(t)$: production for player i at time t.

$I_i(t)$: inventory level for player i at time t.

$I_i(T)$: end-of-year inventory level for player i (at time T).

$\dot{I}_i(t)$: rate of change of inventory for player i at time t.

δ_i : discount rate for player i.

S_i : salvage value of inventory for player i.

$p_{ik}(t)$: price of player i's product in market k at time t.

$C_{ik}(.)$: cost function (of production, processing and selling) for player i to market k.

$G_i(.)$: inventory cost function for player i.

MC_{ik} : marginal cost of production, processing and selling for player i to market k.

h_i : marginal cost of inventory for player i.

$\lambda_i^i(t)$, $\lambda_j^i(t)$: costate/adjoint variable (with respect to the rate of change in inventory) for player i at time t.

γ_{ij}^k : conjectural variation (in quantity) of player i about player j in market k.

ρ_{ij}^k : conjectural variation (in price) of player i about player j in market k.

η_{ii}^k : own-price elasticity of demand of player i's product in market k.

η_{ij}^{k} : cross-price elasticity of demand of player i's product in market k.

$IC_1(t)$: percapita income (disposable) in the U. S. at time t.

$ICG_2(t)$: percapita income (West Germany GDP) at time t.

$IC_2(t)$: the ratio of percapita income (West Germany GDP) to the European currency unit (ECU) at time t.

$AJ_1(t)$: apple juice consumption in the U. S. at time t.

$PAJ_1(t)$: apple juice price in the U. S. at time t.

$ERB(t)$: Brazilian exchange rate at time t.

$SF_k(t)$: Seasonal dummy for Florida in market k at time t.

$SB_k(t)$: Seasonal dummy for Brazil in market k at time t.

TT : time dummy (year).

τ_k : tariff rate in market k.

$INF(t)$: initial inventory level for Florida at time t.

$INB(t)$: initial inventory level for Brazil at time t.

ϵ_{m} : disturbance term of equation m in the quantity game.

ν_{n} : disturbance term of equation n in the price game.

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The competition between Florida and Brazil in two major orange juice markets, the U.S. and Europe, is investigated. The competition is viewed as a dynamic process due to the seasonal nature of production, and a presumed spatial and intertemporal optimization practiced by the market participants. The overall objective of the study is to develop short-run dynamic competition models of the frozen concentrate orange juice (FCOJ) market with particular emphasis given to marketing decisions on quantity and pricing, spatial allocation, and inventory strategies.

Literature related to dynamic optimization and production-inventory theory is reviewed. Relevant theory of industrial organization/economics and the theory of dynamic games are surveyed, emphasizing particularly duopolistic models. Some previous orange-juice-related studies are also reviewed. Different and often conflicting schools of thoughts about imperfect competition, duopolistic competition in particular, are briefly evaluated.

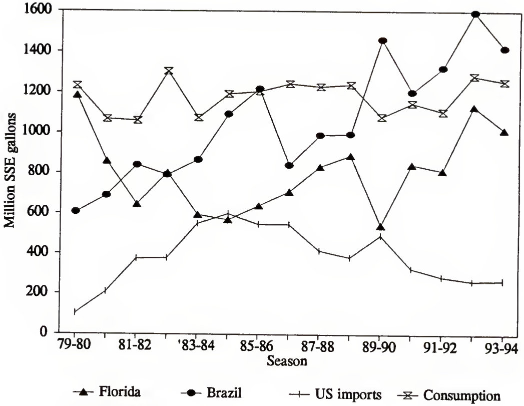
While by no means ideal, a balance between theoretical and practical/empirical purposes has been attempted throughout this study. Two theoretical models, quantity and price games, are developed. A dynamic game-theoretic approach is used in this study to derive analytical solutions of the models, which are formulated as noncooperative differential games, and to characterize the properties of optimal strategies. Due to the structure of the models, which is state separable, the open-loop solutions coincide with the feedback solutions. Conjectural variations were evaluated to derive particular strategies such as Nash and Stackelberg. Explicit solutions were derived based on linear (inverse) demand functions for (quantity) price games. The empirical analysis is carried out to examine market conduct, assuming that a profit maximizing objective was pursued through an integrated spatial and intertemporal pricing and inventory strategy employed by Florida and Brazilian FCOJ processors across a short-run (one year) time horizon. The estimation was based on a three season (1990/91 - 1992/93) sample period. The study shows that the price game model provides slightly better results to portray the competition in the international FCOJ market. Overall, the results show the appropriateness of the approach to the system under study. Rational behavior, that is profit maximization in this case, is an appropriate assumption in investigating the FCOJ market.

CHAPTER 1
INTRODUCTION

Background

One of the important changes in the world orange industry is the emergence of Brazil as a dominant power in the market. The Florida orange industry, which once enjoyed security in its position as the price leader in the world market for orange juice, has been pushed by Brazil. Recent disruptions in the state, due to both nature- and man-induced events, which led to increasing uncertainty in production and marketing complexity, combined with the growth of the market power of Brazil and other producing/exporting countries, has resulted in changes in the competitive and economic environment (see Muraro and Amaro, 1990; McClain, 1989 and Ward and Kilmer, 1989). Figure 1-1 shows Florida and Brazil orange-juice production, and U.S. imports and consumption from 1979 to 1993.

Almost all fresh citrus consumed in the United States is from domestic sources. Approximately 95% of all Florida oranges are processed (FDOC, various issues a). As of 1992-93, the Florida orange industry supplied 93% of the U.S. orange juice production. Severe crop losses from freezes in Florida in late 70s and in 80s enabled Brazil to expand its orange production and supplies which now accounts for about 80% of the world's net orange juice exports (about 29% of which entered the U.S. orange



Source: FDOC.

Figure 1-1 Florida and Brazilian orange juice production, US consumption and imports 1979/80 - 1993/94.

juice market in 1992-1993). As of 1992-1993, Brazil's FCOJ exports to the U.S. were 416.4 million single-strength-equivalent (SSE) gallons, while Florida's FCOJ production (excluding imports) is 842.9 million SSE gallons.

Processors are the major decision-making forces within the Florida citrus industry. They provide most of the producers' pooling arrangements and are directly responsible for pricing and selling processed citrus (Ward and Kilmer, 1989). Their decisions, therefore, are crucial for the entire industry's future development. Processors produce FCOJ (packed in retail, institutional, and bulk forms) from oranges and import additional supplies, mainly from Brazil (in bulk form). Production seasonality and the presence of FCOJ imports are among important factors affecting domestic orange industry.

Production seasonality for fresh citrus can change the relative competitive positions during the year. In contrast, seasonality for processed juices, while important, may be lessened with the continued capability for long term storage and distribution. The opportunity to offset production patterns and to meet seasonal demands is greatly enhanced through storage (Ward and Kilmer, 1989). Economic theory (for example see Samuelson, 1966b; and Blinder, 1982) suggests that when a product is storable, inventory policy may play an important role in determining market outcomes. The relative importance of inventory decisions may depend on, among other things, the nature of the product, market structure and the behavioral pattern of the market participants.

The inventory system may appear as a double-edge decision making instrument. On the one hand, it can lessen the market instability by maintaining 'adequate' goods available to the market. On the other hand, it introduces another dimension of complexity in the market. Since decisions at a particular period of time affect relative positions in the future, decision makers now have to take into account the intertemporal consequences of their decisions. The intertemporal dependence implication of inventory policies adds to market dynamics. All of these demonstrate how crucial inventory policy is in analyzing the FCOJ market. Historically, the industry managed its inventories fairly successfully. Nevertheless, the level of inventories currently maintained may still not be optimal. Recently, the availability of imported juices from Brazil provides a direct alternative in inventory management. In spite of its importance, a direct relationship between imports and inventory management (Ward and Kilmer, 1989) has not been measured. Current inventory decisions may be based on habit persistence rather than a careful evaluation of the market. In an oligopoly market where there exists an import supplier, inventory decisions placed by a foreign producer that has a relatively strong market power in an export market may enhance its own interests but not necessarily be in the best interests of the domestic producers or the aggregate industry.

Benefiting from the decline in Florida's orange production due to severe freezes in the late 70s and in mid 80s, Brazil has been successful in expanding orange juice export markets. Development of export markets is important for the Florida orange industry. Yet the domestic market is at least equally important since Florida's citrus industry must import and compete with Brazilian FCOJ in order to satisfy domestic orange-juice requirements. Increasing competition from the Brazilian industry, coupled with the uncertainty in the state's production and domestic market, creates problems that require more sound decisions. The knowledge obtained from quantitative modeling, when used properly can be very helpful in the decision making process. This study is aimed as an attempt to provide this knowledge.

Current State of the Orange Juice Market

The domestic and international orange juice markets are changing. In the domestic market, among the important changes are the recovery of orange production in Florida and the increasing popularity of pasteurized chilled orange juice. Since most of Florida's processed orange products are marketed in the U.S., it is generally expected that this will reduce the import of Brazilian FCOJ in the future. As indicated by Figure 1-1, orange juice consumption in the U.S. is relatively unchanged and stabilized in the range of 4.3-5.0 SSE gallons per capita in the last five years.

Although the percentage of Florida orange production utilized for FCOJ has tended to decline in the last ten years (in part due to increasing popularity of chilled orange juice), as indicated by Table 1-1, total production has tended to increase (except in the 1989-90 season, due to a freeze). With the estimated increase in Florida orange production in the future, Florida FCOJ will remain very important both for domestic and export markets (for example see FDOC).

On the production side, as reported by Ward and Kilmer (1989), industry structure at the domestic processing level (excluding imported juice entering non-Florida

Table 1-1 Utilization of Florida round oranges (in million boxes).

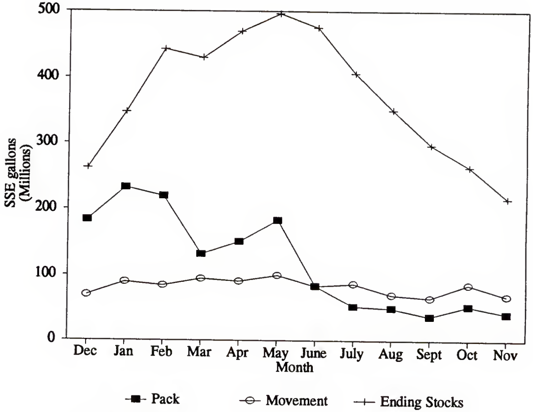
Season	FCOJ	Total	FCOJ as % of Total
1983-84	92.0	119.6	76.9
1984-85	84.1	107.2	78.5
1985-86	91.9	119.2	77.1
1986-87	90.5	119.7	75.6
1987-88	103.9	138.0	75.3
1988-89	107.4	146.6	73.3
1989-90	70.1	110.2	63.6
1990-91	100.4	151.6	66.2
1991-92	90.6	139.8	64.8
1992-93	128.3	186.5	68.8

Source : FDOC, various issues a.

ports) is oligopoly with relatively low industry concentration (Herfindahl index (HI) averaging at 0.0543). The largest firms' growth has been such that their shares of processed output have been relatively constant. Individual firms' shares are low relative to imported juice (from Brazil).

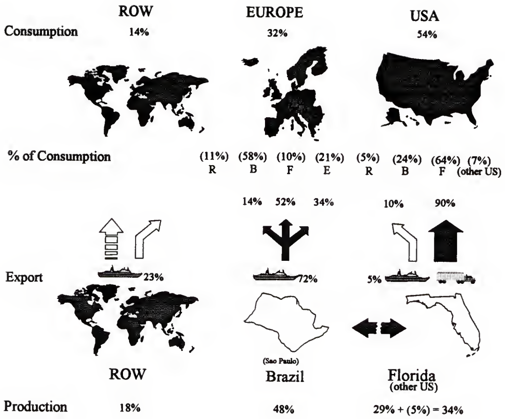
Figure 1-2 shows the Florida FCOJ (total of bulk, retail and institutional) situation in 1992-93 season. Monthly ending inventories seem related to production seasonality, while movement is relatively stable throughout the season. FCOJ inventories start to increase as the early, mid-season, and navel varieties, which account for about 57 to 60 percent of the total Florida round oranges (FASS, 1994), is harvested from October until early March. The peak of the inventories is reached when the remaining output, comprised of later maturing Valencia oranges (noted for high juice content), is harvested around June.

In the international market, Brazil continues to be the dominant orange juice supplier. Figure 1-3 is a *snapshot* of the recent world orange juice situation. There are three major orange juice producers/exporters--Florida (plus other U.S.), Brazil, and the rest of the world (ROW)-- and three major consumers (the U.S., Europe, and the rest of the world). According to FAO and Florida Department of Citrus data as of 1990-91, Brazil's production accounts for 48%, Florida (other U.S.) accounts for 29% (5%), and ROW accounts for 18% of world orange juice production. About 96% of Brazil's orange juice production is exported, which accounts for about 72% of the world orange juice exports in that year. Of this amount, approximately 34%, 52%, and 14% are exported to the U.S., Europe, and ROW, respectively. In contrast, about 90% of Florida's orange



Source: FDOC.

Figure 1-2 Florida FCOJ situation in 1992-93 season.



Source : FAO, and FDOC.

Figure 1-3 The world orange juice situation in 1990-91.

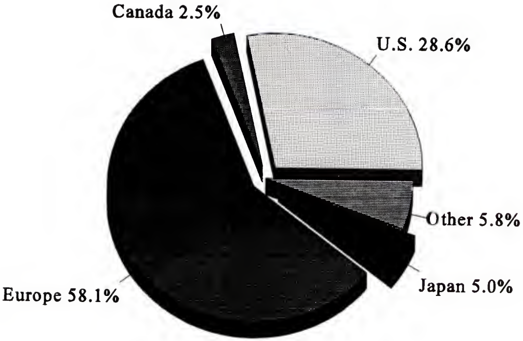
juice production is for U.S. domestic consumption, and only about 10% is exported. U.S. orange juice export (mostly from Florida) accounts for only about 5%, while ROW export accounts for about 23% of the total world orange juice export in that year.

On the demand side, the U.S. is still the largest market for orange juice and accounts for about 54% of world consumption (1990-1991), while Europe and ROW account for about 32% and 14%, respectively. Of total U.S. orange juice consumption that year, 64% is supplied by Florida, and 7%, 24%, and 5% are by other U.S. producers, Brazil, and ROW, respectively. These numbers show the importance of Florida and Brazil in the domestic orange juice market.

In Europe, only about 21% of the total consumption is fullfilled by the European countries' production. About 58% is imported from Brazil, 10% is from the U.S. (mostly from Florida), and about 11% is from ROW. Figures 1-4 and 1-5 demonstrate Brazilian FCOJ export markets and the domestic situation.

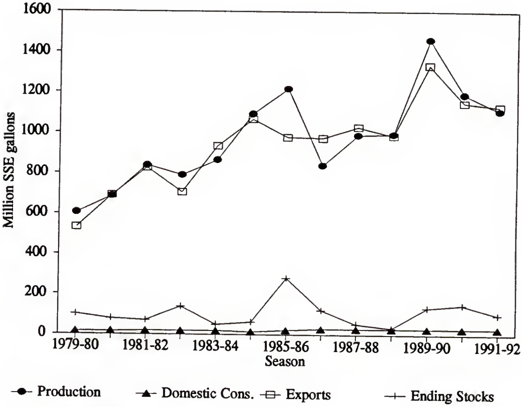
Problem Statement

Brazil and Florida are the two major producers of orange juice in the world, accounting for about 85% of total production. Europe and the United States, in the meanwhile, continue to be the two largest orange juice markets in the world. Instability in orange juice market is affected by various factors including fluctuation in production, inventory strategies, and trade policies.



Source : FDOC.

Figure 1-4 Brazilian FCOJ exports by destination in 1992-93.



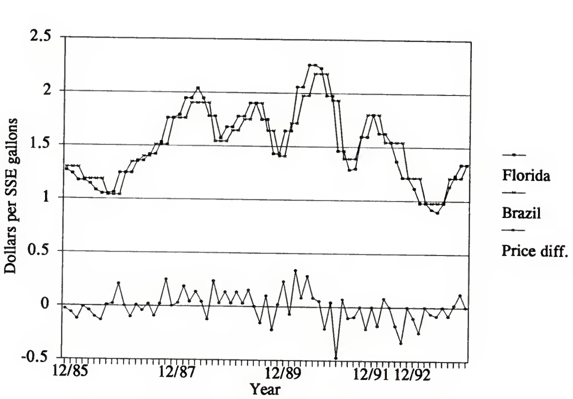
Source: FDOC.

Figure 1-5 Brazilian FCOJ situation in 1979-92.

> According to Ward and Kilmer (1989: p. 55), if inventories tend to accumulate faster than the norm, the industry must initiate corrective marketing programs. The citrus industry, under the present market structure, has three major options available to deal with large inventory accumulations. Stepped-up branded and generic advertising programs designed to shift the demand for orange juice may be undertaken. Price cutting at the FOB level can also be used to reduce accumulating inventories. However, because of the inelastic demand for frozen concentrate, this policy also tends to reduce the current season net dollar gain to the industry. Finally, new inventory policies may be implemented, with the amount of FCOJ carried into the next crop year increased. Unlike during the early seventies when Florida processors supplied almost all orange concentrate, processor storage policies must now include an evaluation and speculation about Brazilian supplies. Clearly the value of stored concentrate is directly impacted by Brazilian crop sizes and pricing policies.

Unlike Florida orange production, which are harvested from October to June (marketing season of FCOJ begins in December), Brazilian oranges are harvested from May to the next April. The FCOJ marketing year in Brazil begins in July and ends the following June, with peak marketing to the U.S. usually from September to November. Therefore, imports of FCOJ from Brazil are typically available at the beginning of Florida season. With the expectation of increasing orange production throughout the world and relatively slow growth of demand, competition in the export market can be expected to put further downward pressure on prices as Florida orange production recovers from the freezes of the 1980s.

One of the primary concerns in the orange market is variation in price and production. Instability in the market may be beneficial to some individual producers in the short-run, but will most likely be detrimental to the state's industry in the long-run. Figure 1-6 demonstrates how the FOB price of Florida and Brazilian bulk FCOJ changes



Source: FDOC.

Figure 1-6 FCOJ prices in the U.S.

over time. An increase (decrease) in Florida bulk FCOJ FOB price is almost always followed by an increase (decrease) in Brazilian bulk FCOJ FOB price, and *vice versa.*

In the U.S., transactions may be accomplished through a few large processors who are of sufficient size to have substantial control over the industry (Ward and Kilmer, 1989). The ability of a few firms to exercise their market power through pricing and other marketing policies suggests an oligopolistic structure. It is generally argued that there exists strong price leadership among processors in the FCOJ market. Recognition of imperfect market competition implies an existence of interdependences among the producers. It is unreasonable, therefore, to assume that the market participants act freely as a monopolist nor act so passively as in a perfect competition.

The dynamic nature of the problem is an important aspect that cannot be ignored. This is, in particular, not only due to these interdependences, but also the existence of intertemporal dependence in the system; any decision/strategy implemented by Florida or Brazil in a particular period affects their positions in the future. In addition, each of them has to take into account the spatial dimension of their decisions due to different target markets. Hence, one of the problems in this dynamic-competitive environment faced by processors is resolution of the conflicting objectives of maximizing net present revenues, determining spatial allocative decisions, and controlling inventories such that sufficient supply is available in the future.

A research effort related to the total industry (and not to specific firms) is important as it will have its greatest usefulness in providing economic impact studies for various pricing and inventory policies. Previous orange-juice-related studies placed emphasis on the long-term horizon. Yet it is obvious that the short-run (*i. e.*, one season year) strategies are crucial to the citrus industry. The expected benefits of a better strategy merit the investigation on a short-run dynamic inventory and pricing model of the system.

Objectives

The purpose of the study is to investigate the short-run dynamic competition in the orange juice market. The specific objectives of the study are

(1) to develop a short-run dynamic duopoly game model of the frozen concentrate orange juice market,

(2) to provide a formal analysis and examine optimal strategies of product pricing, spatial allocation, and inventory system, and

(3) to demonstrate the use of the model in analyzing the degree of competition in the present structure of the international orange juice market.

Although retail and institutional FCOJ are important, most of Florida and Brazilian FCOJ is in bulk form. It is expected then, that the *intense competition* between the two producers is on the bulk FCOJ. Hence, the central attention of the study is on the bulk FCOJ market.

Methodology, Scope, and Organization of the Presentation

In competitive markets, rivals' reactions are not at issue since the market participants are tied together by parametric pricing. At the other extreme, under a monopoly structure, single or joint decision making makes competitors' reactions unimportant. Microeconomic theories about these two polar market structures are well-developed. Between these two structures, imperfect market structures are common if not ubiquitous in the real world. Yet economists have only proposed a number of competing hypotheses on how firms behave in these settings. A perfectly determinate and general theory of imperfect competition is out of question, however, because as many studies show, each industry presents its own problems and characteristics.

The traditional *structure-conduct-performance* (SCP) paradigm in industrial organization potentially provides a rich and flexible framework for market analysis to help understand the complexity of reality in this study. Since modeling of the problem facilitates organization of thoughts and the formulation of testable hypotheses about how markets work, theoretical models will be first developed and investigated to provide foundations for the next step, the empirical analysis.

The competitive behavior of firms in imperfect markets is a question of fact and thus must finally be resolved by empirical testing. The issues in this dissertation will be confined to the levels of competition in price and quantity. The analysis is limited to frozen concentrate orange juice at the industry level (not to specific firms) where Florida and Brazil are assumed to be the only *'players'* in the system under study. Further, the market competition is considered to be a dynamic process, and hence the structure and solution concepts of the game theory approach are used to investigate it. More specifically, the research emphasis is on developing a formal analysis utilizing dynamic duopoly games models to examine the problem.

The organization of the dissertation is as follows. In Chapter 2, the literature related to dynamic optimization and production-inventory, industrial organization (especially related to duopoly competition models), and dynamic games is reviewed. A review of selected previous studies related to domestic frozen concentrate orange juice is also presented. In Chapter 3, the proposed models are developed. Chapter 4 is comprised of discussions on derivation/characterization of analytical results/solutions and their policy implications in the model. In Chapter 5, numerical results are presented and discussed. Chapter 6 summarizes the study and presents the conclusions and suggestions for further research.

CHAPTER 2
REVIEW OF LITERATURE AND THEORETICAL FOUNDATIONS

There are several previous important works relevant to the issues and methodologies in this study. The review presented here, however, will be limited especially to those that have significant relevance to this dissertation. The organization of this chapter is simply to ease the discussion and not intended as a scientific classification of knowledge.

Dynamic Optimization and Production-Inventory Theory

Calculus of variations, optimal control theory (OCT), and dynamic programming (DP) are optimization techniques commonly employed to solve various types of intertemporal optimization problems where traditional static optimization techniques may not be appropriate. Aside from the debate whether or not *optimizing behavior* is a correct assumption of economic agents, these dynamic optimization techniques have become more popular as today's problems are also increasingly complex. The tools are now nearly standard in economics and management science/operations research. Bellman and Dreyfus (1962) and Dreyfus and Law (1977) are among popular texts in DP. While Kamien and Schwartz (1991), Leonard and van Long (1992), and Chiang (1992) are excellent texts for calculus of variations and OCT.

Perhaps, the perceived strength of optimal control in agricultural economics is in stipulating the *best decision* given the objective function and constraints of the economic system. Zilberman (1982) argued that since OCT is a powerful tool which expands the range of issues dealt with by agricultural economists and increases their effectiveness, it should become part of the mainstream tools of agricultural economics. He asserted (1982, p. 397): " the controversies in agricultural economics are mostly with regard to the exact objectives of policymakers and the constraint structure they face. Therefore, there is a higher premium to models that better clarify objectives and, in particular, constraints of the system than to models that obtain better numerical results for systems based on shaky ground."

Dynamic optimization, in fact, gains popularity among agricultural economists, *e.g.*, Hochman *et al.* (1974), Rauser and Hochman (1979), Kennedy (1986), and Gao *et al.* (1992). Hochman *et al.* (1974), for example, developed an OCT model to explore an effective advertising policy for the Florida citrus industry. Rausser and Hochman (1979) emphasize the conceptual and technical aspects of OCT in the context of dynamic economic systems, especially agriculture. Kennedy (1986), discussed DP and its application to agriculture and natural economics. Gao *et al.* (1992) developed a Markov decision bio-economic model of the breeding and replacement decisions in dairy production. The model was solved via stochastic dynamic programming and used to analyze the potential supply response to seasonal price premiums.

Discussions on the production-inventory subject are unseparated from dynamic optimization. One of the earliest and best-known studies in production-inventory planning is the research of Holt *et al.* (1955, 1960). Their model basically minimized a quadratic cost function (to represent U-shaped average cost curve) subject to a multiperiod inventory constraint. They solved their production-inventory model using the calculus of variations technique. Since the decision rules resulting from the solution are linear functions of the forecasts by period over the planning horizon and the current net inventory and work force levels, the model is called the linear decision rule model. Johnson and Montgomery (1974, Chapter 4) provide a good introduction to the application of the DP technique to production-inventory systems. Bensoussan *et al.* (1983, Chap. III) demonstrate the use of OCT for production-inventory problems.

There are various motives holds that inventories of finished goods (see Williams, 1984). There is the transaction motive (derived from the Keynesian argument for holding money): that is the amount held for the need of daily economic activity. It is an intermediate stage between production and consumption. The speculative motive holds that the price of goods held in inventory may be expected to rise. Buffer stock is held as a safeguard against emergency or due to the possibility of incorrect sale forecasts. Production smoothing provides for an adjustment in production in anticipation of the change in sales or in production level. Some also argued (*e.g.*, Rotemberg and Saloner, 1989) that in an imperfect market competition, the inventory decision may be used as a firm's policy instrument to deter deviations from an implicitly collusive arrangement. Higher inventories allow firms to punish cheaters more strongly and can thus help to maintain collusion. Various rules of thumb (such as the proportional rule, square root rule, and range rule) were also developed in solving inventory problems that lead to different conclusions.

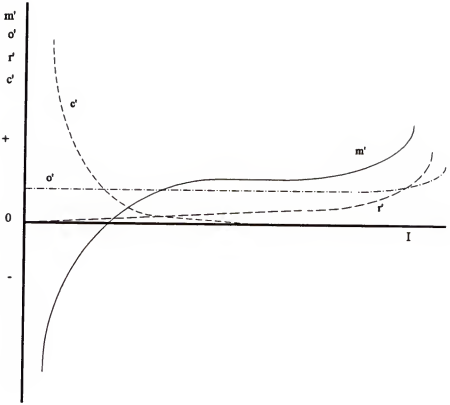
Most of the production-inventory models were developed either in a monopoly or perfect competition framework. Pekelman (1974), for example, considered production rate and price as decision variables in a production-inventory problem of a monopolist firm. He treated the optimal control problem of simultaneously determining the optimal price and production rate of a firm over a finite planning horizon by setting the firm's net discounted return as the objective function and assuming that the firm faces a time-dependent linear demand function, convex production cost and linear inventory cost. The model enforced a nonnegativity state constraint for inventory level.

Feichtinger and Hartl (1985) extended Pekelman's model by considering general (nonlinear) demand functions and relaxing the nonnegativity constraint for inventory by allowing for shortages that are penalized by shortage costs. Again, their model is based on a monopolistic framework, where demand is determined by the price charged by the firm. They solved the problem by applying the necessary and sufficient optimality conditions of *Pontryagin's maximum principle* and showed the existence of a unique equilibrium. This point is reached within finite time if the inventory and shortage costs are linear although the model is nonlinear in both control variables (production and price). This is a consequence of the nondifferentiability of cost functions, which implies that there is no equation (only an interval of possible values) for the shadow price associated with a stationary shortage level, whenever the state variable (the inventory level) is zero.

Among the early studies related to inventory theory in economic literature is that of Working's *theory of price of storage* (1949). He emphasized the importance of inter-temporal price relations (cash and futures markets). They are viewed as one market for the same good across time. Stock level is the key in determining the price spread. The return for storage (which is also the 'price of storage') is the principal means by which futures markets facilitate the economical distribution of supplies through time.

Brennan (1958) showed that the determinants of the futures price are also applicable in market with no futures market. He argued that in a competitive industry, a firm seeking to maximize net revenue will hold an amount of stock such that the net marginal cost of storage per unit of time equals the expected change in price per unit of time. On the supply side, in addition to the marginal expenditure on physical storage and the marginal convenience yield another variable, a risk premium, is required to explain the holding of stocks as a function of price spreads.

Brennan defined the net marginal cost on physical storage plus a marginal risk-aversion factor minus the marginal convenient yield on stocks. The total cost on physical storage is the sum of rent for storage space, handling or in-and-out charges, interest, insurance, etc. Similar to Working's marginal cost model, he suggested that the marginal cost is approximately constant until total warehouse capacity is almost fully utilized, beyond which marginal cost will rise at an increasing rate. Figure 2-1 shows the net marginal cost of storage (m') and its three components for a typical firm according to Brennan: marginal cost of physical storage (o'), marginal risk-aversion factor (r'), and marginal convenient yield (c'); where I is the level of inventory. According to Brennan,



(source: Brennan, 1958).

Figure 2-1 Marginal cost of storage

the convenient yield is attributed to the advantage (in terms of less delay and lower costs) of being able to keep regular customers satisfied or of being able to take advantage of a rise in demand and price without resorting to a revision of the production schedule.

By assuming that future events were known with certainty, Samuelson (1966b) developed a theory of storage and temporal pricing. He considered a competitive market of an agricultural commodity. The analysis demonstrates the accumulation of stocks and price declines at harvest time, and then price rises seasonally until the next harvest season. If positive storage is worthwhile, price must be rising at a rate given by time storage costs. As he explained that economic relations in *time* have many properties of economic relation in *space*. In two competitive markets separated by space (see Samuelson, 1966a), the *price differential* (the difference between the prices in the two markets) equals the transportation cost between the two markets. If two stylized competitive markets are separated by time, the *price spread* (the difference between the prices at different points of time) equals the transportation cost for transporting the commodity from one time to a later future time, *i.e.*, the storage cost.

Helmberger and Weaver (1977) developed a theory of storage under conditions of uncertainty assuming that producers and arbitragers are guided by rational expectations. Following the work of Samuelson (1966b), Turnovsky (1974), and Helmberger and Weaver (1977), Helmberger *et al.* (1982) developed a theory of competitive pricing and storage under uncertainty. Their model was applied by Mataya (1983) to analyze Florida FCOJ storage levels.

Knapp (1982) provided a graphical analysis of optimal storage, trade, and borrowing policies for grain using dynamic programming. The model assumed stochastic world prices and domestic harvests. He found that optimal carryovers are generally increasing in supply and decreasing in world prices, while net imports are decreasing in both supply and world prices. He showed that grain reserves and foreign exchange borrowing/saving were alternative inventory systems which both complement and substitute for one another.

Phlips (1980) attempted to show that when the optimization is done in an intertemporal framework, price rigidity is compatible with optimizing behavior. More specifically, normal cost pricing is seen to lead to decision rules compatible with those resulting from intertemporal price discrimination. In addition, the possibility of building up inventories leads to the intertemporal price discrimination rule.

Blinder (1982) contributed a theory of inventory and sticky price as microfoundations of macroeconomics. He indicated that for given marginal cost and marginal revenue curves, *'sticky'* prices will tend to emerge when it is not very costly to vary inventories and when demand shocks are transitory. He summarized some interesting conclusions. When output is not storable, firms can react to an increase in demand only by raising prices or by increasing production. The more they do one, the less they have to do of the other. When output is storable, however, the same firms that sharply raise prices may also increase production, while other firms may raise both price and output little. Firms in the latter category will take up the slack by extensive reduction of inventories. Firms, whose marginal costs of inventory holding are relatively constant and whose demand shocks are transitory, will rely heavily on inventory changes to absorb shocks and will adjust price and output little. Conversely, firms with sharply rising marginal inventory costs and/or permanent demand shocks will rely less on inventories as buffer stocks and will exhibit larger price and output fluctuations. Furthermore, the reactions of output, price, and sales to inventory stocks are strongest when the firm's inventory carrying cost function is most convex, weakest when it is close to linear.

The work by Samuelson (1966a) is among the early references in spatial price equilibrium models. Together with his intertemporal price equilibrium model mentioned earlier, his works are clearly important for the development of the spatial and intertemporal optimization models. Intertemporal and spatial optimization models were later discussed by Takayama and Judge (1971, 1977) and Takayama (1977). The complexity of the models increases as time and space dimensions are considered simultaneously. Takayama and Judge (1971) discussed in considerable detail the use of the framework of mathematical programming as a basis for specifying alternative models that may be used to evaluate alternative admissible spatial and intertemporal pricing and allocation systems and decisions. They developed intertemporal, spatial, and combined intertemporal and spatial equilibrium models for both monopoly and competitive firms, and then utilize Lagrangian techniques and the Kuhn-Tucker conditions to derive optimality conditions to solve the mathematical models. Among the important results of their analyses (for the single product case), the standard equilibrium/optimality condition may be summarized as follows.

It is assumed that the net social welfare to be maximized is the *Pigouvian* type or additive, and the inverse regional market demand has the usual properties of continuous, differentiable and monotonically decreasing of the consumption quantity. The supply function in any region is assumed to be continuous, differentiable and monotonically increasing of the production quantity. The competitive equilibrium has the property that the prices in two regions only differ by the transportation costs between those regions (assuming that unit transportation cost is independent of volume).

Similar to the spatial models of Takayama and Judge, McCarl and Spreen (1980) discussed a price endogenous model for a sectoral analysis. The *sector activity* dimension in their model is essentially analogous to the *space* dimension in Takayama and Judge's spatial model. 'Spatial' models are also the central subject investigated by Greenhut *et al.* (1987). The main feature of their analysis is the power (which rests in the ability of the producer to assign its customers to *separate* or *differentiate* markets) that individual producers have to set prices. They argue that a spatial framework opens up entirely new ways of viewing long-standing problems in macroeconomics. For example, the pricing of services or of differentiated goods in general, even though not distance-related *per se*, can be analyzed from a spatial perspective. The framework is formulated on the presumption that producers sell to spatially separated and distinguishable markets, each typically subject to different demands and competitive influence. Thus, the recognition of market discrimination brings with it the need to develop serious insights into imperfect market structures.

Industrial Organization/Economics and the Theory of Duopoly

Studies in industrial organization (IO) flourished based upon the less-developed microeconomic theory of imperfect competition. Unlike the extreme cases of perfect competition and monopoly (where each seller finds its best decision by straightforward optimization), imperfect competition markets are characterized by nonegligible interdependencies among market participants. In these types of markets in general, the decisions made by a firm (quantity/output, product quality, advertising, pricing decision, etc.) depend on the decisions of its rival. In addition, the interdependence is revolving: a firm's decisions depend on its rival's decisions, which in turn also depend on its rival's decision, and so on. Even in a given competitive situation, it is extremely difficult to derive an unambiguous equilibrium. There may be several different and entirely reasonable courses of action open to firms in selecting a competitive strategy. Thus, unsurprisingly varied and often conflicting schools of thoughts can be found in industrial organization/economics literature.

Of the various theories of industrial organization, perhaps the first and most influential is that associated with the pioneering work of Edward S. Mason and Joe S. Bain, which sometimes called the *'Harvard tradition'* (Tirole, 1988; and Reid, 1987). The traditional approach to the study of markets in industrial organization has been based on the classical *'structure-conduct-performance'* (SCP) paradigm. It postulates that market structure (the number of firms in the market, the degree of product differentiation, etc.) determines the conduct/behavior of firms (profit maximizing, selection of the strategic variables, cooperative or noncooperative competition, etc.), which eventually determines market performance (profits, efficiency, innovation, etc.). Such a study is typically aimed at describing the form of competition observed in the market. One of the criticisms to the approach is that (Tirole, 1988), although the paradigm is plausible, it is often based on loose theories, more of the case-study approach, and the emphasis is placed on empirical measurement.

In comparison with the *Harvard tradition*, the work of Aaron Director and George J. Stigler (the *'Chicago tradition'*) emphasized the need for rigorous theoretical analysis, which had an important methodological impact on the development of the field. Reid (1987) reviews some popular schools of thoughts in IO. The more recent development in IO is presumably inclined to the development of more rigorous theoretical works (see for examples modern texts in IO of Tirole, 1988; and Krouse, 1990). IO is a fast growing field of economics. It is impossible to review numerous important and relevant previous works here. The following is a brief review of literature (related to the theory of duopoly) in quantity and price competition and collusion models.

Quantity and Price Competition Models

Duopolistic competitions (competitions that involve two firms/players) are probably the most widely studied of oligopolistic models. The standard model of duopoly is that of Cournot. The model assumes a linear demand curve for a homogeneous good and begins with zero marginal cost. It is also assumed that firms/players commit themselves to *quantities* and then let the market determine the market-clearing price. There are at least two interesting points of Cournot equilibrium. First, the model provides 'predicted' solutions that are between the monopoly and the competitive solutions. Second, when entry is allowed, it reduces to competitive model as the number of players goes to infinity, and it reduces to monopoly when there is only one player. In addition, the equilibrium quantity increases as the number of players increases.

Bertrand (see Friedman, 1983) argued that in many situations, sellers/players commit to *prices* and then adjust their rates of production to fit consumer demand at those prices. It is assumed that when players produce a homogeneous product, consumers care only about the price in choosing between one player and another. This model is also appealing and leads to a complete turnaround in market prediction compared to Cournot model. For example, given any price above marginal cost charged by one player in a duopoly, the other player can increase its profits by lowering its price and taking all the business away from the other player. If there is no player with a capacity constraint, this process continues as long as price is greater than marginal cost. The (Nash-Bertrand) equilibrium is achieved when price equals marginal cost, at which neither player has an incentive to lower its price because it would result in negative profits on the marginal units. Bertrand, therefore, argued that the competitive solution is the appropriate equilibrium even with only two players.

Nash extended the Cournot model by assuming that players take their rivals' outputs/strategies as given. A Nash equilibrium refers to the optimal strategy for a player given the strategies of all other players. The Cournot model has been criticized (for example, see Bresnahan, 1981; and Kamien and Schwartz, 1983) for assuming that its decision/strategy does not affect the other player's strategy. In reality, a player may expect its rivals' behavior to change in response to the player's decision. Thus, we may expect the player to plan sequences of moves in response to conjectures about how their rivals will respond, which is usually called conjectural variations (CV). According to this formulation, the zero conjectural variation assumption in Cournot model is inconsistent. The supporters of Cournot model (for examples, see Daughety, 1985; Lindh, 1992; and also Krouse, 1990), however, argued that the Nash-Cournot equilibrium is consistent, and provide some theoretical and empirical objections to the conjectural variation model.

Stackelberg explored the implications of alternative behavioral assumptions for the duopoly model. The Stackelberg model assumes a *relative position* in the market: one player acts as a *leader* (or dominant player), and the other player is called the *follower*. In the original Stackelberg model, the leader sets price by choosing a profit-maximizing quantity over the difference between market demand and the quantity offered by the other player. The follower takes the price set by the leader as given and behaves competitively. It may be concluded that in general (see Appendix A for mathematical derivations): (i) pure quantity strategy oligopoly almost always has a "leader-leader" outcome (*disequilibrium* Stackelberg), (ii) pure price-strategy oligopoly cannot have a "leader-follower" outcome, as shown by Bertrand that the outcome must be competitive price. The endless debates among various models supporters are apparently rooted back at differences in developing the appropriate and acceptable assumptions underlying the models and their *solution concepts*, and in their personal preferences. As argued by Krouse (1990), the process to derive an equilibrium can be viewed as a pre-market process and does not describe an actual sequence of behavior. He asserted that this pre-market process is needed to provide an analytical procedure for deriving the equilibrium, and thus any perverse behavior does not invalidate a particular equilibrium (Nash-Cournot, for example) as a *solution* (or *equilibrium*) *concept*.

Figure 2-2 portrays a Cournot duopoly of a homogeneous product with a linear demand curve (and hence the duopolists' reaction curves, r_1c and r_2c, are also linear). Reaction curves indicate the *best reply* (for example, based on profit maximizing criterion) in term of the amount/quantity by which one of the duopolists will change its output in reaction to a change in the amount produced by the other. Tangent to the curves r_1c and r_2c are, respectively, the isoprofit curves for player 2 and player 1. Point C is the Nash-Cournot equilibrium. S_i is the Stackelberg equilibrium when i is the leader, and S_d is the Stackelberg disequilibrium when both duopolists assume the other as the follower. For more detail of the mathematical derivations of Cournot, Bertrand, and Stackelberg equilibria, see Appendix A.

Among the empirical studies of imperfect market models related to agriculture is Kolstad and Burris (1986). They hypothesized four different market conducts in the international wheat market (free trade, duopoly, triopoly, and duopsony). Their model assumed a Cournot game, *i.e.*, the decision makers choose the quantity as their strategy/decision variable. By assuming the Nash-Cournot as the solution concept, they derived the equilibria for the four models. They found that among the four models, the Nash-Cournot solution for the duopoly and triopoly models performed considerably better, with the duopoly model forecasts being slightly closer to the actual values than the triopoly model.

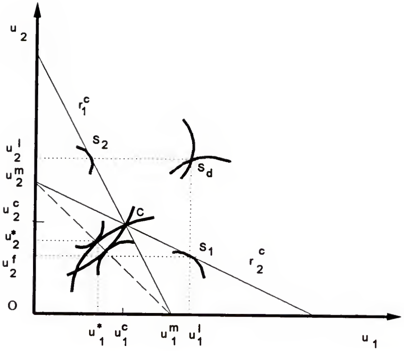


Figure 2-2 A Cournot duopoly.

In many cases, as Bertrand argued, price instead of quantity, is probably the most appropriate decision variable. Thus, decision makers essentially face a pricing decision problem. Figure 2-3 shows a Bertrand duopoly, again with a linear demand curve of a homogeneous product. Curves r_1c and r_2c are the price reaction functions of players 1 and 2, respectively. Curves 1 and 2 are, respectively, the isoprofit functions for players 1 and 2. Point B is the Nash-Bertrand equilibrium. S_i is the Stackelberg equilibrium when i is the leader. A pure Bertrand duopoly, however, has been critized as a misspecified model due to the possibility of contradictive results (competitive price and the absence of an equilibrium).

Variations of the Bertrand analysis may take different paths by relaxing the important underlying assumptions of the model. First, as proposed by Edgeworth (see *e.g.*, Kamerschen and Valentine, 1977; and Krouse, 1990), who tried to solve the 'paradox' of the competitive price result of Bertrand model by imposing capacity constraints to the firms. Thus, the Edgeworth duopoly model may be considered as a short-run version of Bertrand model. Figure 2-4 illustrates the model. Assuming zero marginal costs and proportional rationing with a linear demand curve, the market is divided equally between the duopolists A and B (AC is the demand curve for A, and BC is demand curve for B). OD is the maximum quantity that A can produce, and OE is the maximum quantity that B can produce (assume OD = OE). The monopoly price p^m would be chosen if the two firms were to collude. Hence, A would supply OJ, and B would supply OK (JK = ½ AB). If collusion is ruled out, each firm would then lower its price down to its competitive level (where the price equals its marginal cost, *i.e.*,

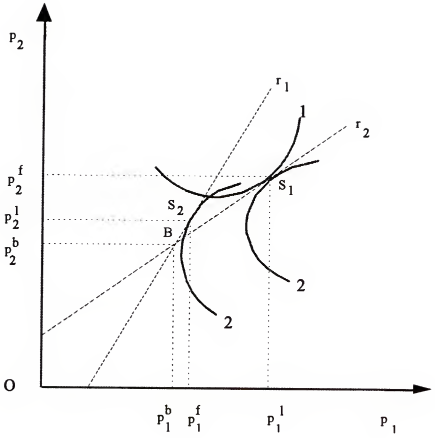


Figure 2-3 A Bertrand duopoly.

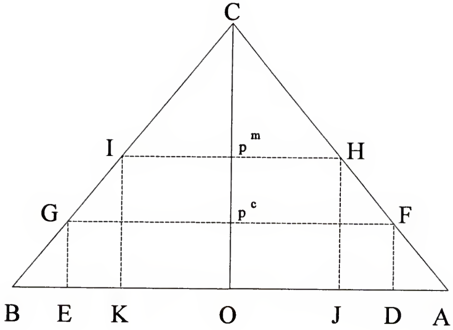


Figure 2-4 Edgeworth duopoly model.

zero). Due to the capacity constraint, however, each firm would only be able to supply up to its capacity, where the price equals p^c. Yet price p^c is not a stable solution either since one firm now could raise its price and increase its total revenue. So long as B maintains its price at p^c, its sales would be at its maximum output level, and A could raise the price without losing customers to B. Edgeworth's analysis then, provides an indeterminate solution with price oscillating without an end between p^m and p^c.

The second path to extend the original Bertrand model may be taken by introducing temporal dimension into the model and the possibility of the rival's reaction (see Tirole, 1988). The threat of future losses in a price war leads to various types of price leadership and tacit/implicit collusion models are among the results of this relaxation.

Third, by relaxing the assumption of perfect substitutability of the firms' products, as developed by Chamberlin. When products are perfect substitutes for each other, consumers are indifferent between the goods at equal price and, thus buy from the lowest-priced producer. The effect of product differentiation is essentially that the producers have some discretion in the determination of the price. Hence, they can exercise their market power, and not act as price takers. Aside from the various criticisms to Chamberlin's model, his main contribution is the introduction of product differentiation and of selling strategy as two additional policy variables in the decision-making process of the firm.

Conjectural Variation (CV) Models

An important characteristic of duopoly markets is the presence of interdependence between the two firms (or among firms, for oligopoly). The idea of *conjectural variation* (CV) models (named by Frisch; see Friedman, 1983; Kamien and Schwartz, 1983; and Iwata, 1974) is to develop explicit conjectures or prediction about how rival firms will react to its own decisions/strategies. Conjectural variations are often used to characterize the degree of competition in oligopoly markets. Some, however, prefer to use them as behavioral parameters.

Bresnahan (1981) proposed a consistency condition (consistent conjectures equilibrium/CCE) that can be used to determine specific conjectured reactions (and thus narrow the possible market equilibria). Perry (1982) explained that in a quantity duopoly game case, a conjectural variation by one firm about the other firm's response is consistent if it is equivalent to the derivative of the other firm's reaction function with respect to the first firm's output at equilibrium. As shown by Kamien and Schwartz (1983), the type of the demand function, the cost function, and the choice of the decision variable are crucial in determining the existence of consistent constant conjectural variations (CCC). They derived CCC for linear and loglinear demand functions. For the differentiated product case, a CCC exists for both types of demand functions when quantity is the decision variable. A CCC, however, only exists for the linear demand when price is the decision variable. The difficulty of finding consistent conjectures makes the issue unattractive to some, and thus it is rarely imposed as an important condition.

Cheng (1988) considered a domestic market that is served by a domestic firm and a foreign firm to derive optimal trade and industrial policies. He developed a static model using CVs to represent different forms of duopolistic competition resulting from the adoption of different schemes (especially Bertrand and Cournot competitions). The model maximizes the social welfare of the domestic country as a function of consumer's surplus, the domestic firm's profit, and government revenues (tariff minus production subsidies). He found that the optimal policy combination can be quite sensitive to the nature of the duopoly's competition.

Among the empirical studies applying the conjectural variation concept are Iwata (1974), Rogers (1989), Liang (1989), and Suzuki *et al.* (1993). By estimating the conjectural variations, Iwata investigated competition in the Japanese flat glass industry. He assumed constant elasticity market demand, marginal cost, and constant conjectural variations. Assuming profit maximizing behavior and noncooperative competition among the firms, the empirical results showed, that the Cournot conjecture hypothesis cannot be rejected and that there might have existed some collusive behavior in the industry.

Rogers (1989) estimated the conjectural variations to study the competition among the eight largest U.S. steel firms. His empirical results also show a mix of different behavior across firms. Different assumptions about the output/cost elasticity also lead to significantly different values for some firms' conjectural variations. He concluded that neither competitive behavior nor Cournot behavior can be definitely proven or refuted. In addition, the collusive behavior hypothesis in the U.S. steel industry is definitely rejected.

A CV model with price as the decision variable was applied by Liang (1989) to measure the degree of price competition in the breakfast cereal industry. The empirical results showed that competitive pricing behavior hypothesis is rejected in favor of collusive pricing. It is concluded that the hypothesis of a unique consistent conjecture is also rejected. She further suggested that consistency may not be a good predictor of behavior in oligopoly models.

More recently, Suzuki *et al.* (1993) estimated conjectural variations to determine the degree of imperfection in the Japanese milk market. They found the change in the Japanese milk market from imperfect competition to a more competitive market and showed the policy implication (*i.e.*, price support) under the two different market structures.

There are criticisms to CV models. First, that each firm's conjecture about the output response of the rival firms would not be confirmed if such a firm altered its output level from the equilibrium. Secondly, there is no guarantee for uniqueness of CCE when the demand function is nonlinear (see *e.g.*, Laitner, 1980; Robson, 1983; and Kamien and Schwartz, 1983). Furthermore, as shown by Kamien and Schwartz (1983), a CCC does not exist when price is the strategy/decision variable even for the most-widely-used loglinear demand function (a constant conjecture variation in price exists but it will not itself be consistent). Some empirical studies discussed earlier also indicate that applications of conjectural variation concept gives unsatisfactory results. Friedman (1983) also criticizes the misapplication of CV in static models, since the concept of CV is basically applicable only in a dynamic sense and not static.

Collusion Models

Previous duopoly or oligopoly models seem to imply that price stability requires some form of *collusion*. Among interesting (and of particular importance) collusion models relevant to this study are tacit (implicit) price collusion models. Many authors observed that in oligopoly markets, whenever competing firms are selling essentially homogeneous goods, competitive forces will eventually drive them to sell at the same price (*e.g.*, Ono (1978, 1982); and Thompson, 1989). One earlier version of the price collusion argument is based on the kinked demand curve theory of Sweezy (*e.g.*, Stigler, 1947; and Call and Holahan, 1983). The theory posits asymmetrical response to the changes in price, where price reduction will be followed by rival firms but price increase will not. The theory implies the price rigidity/stickiness in oligopolies. Stigler (1947) later on attacked both the theoretical and empirical foundations of the theory. He then used price leadership concept to further test the implication of the kinked demand theory. Stigler (1964) proposed a joint profit maximizing motive to explain collusion in his theory of oligopoly (that is, the combined profits of the entire set of firms in an industry are maximized when they act together as a monopolist). Due to an incentive for firms to deviate from the collusive price, the monopoly results will never be achieved perfectly.

There are at least four types of price leadership found in the literature (*e.g.*, Kamerschen and Valentine, 1977; Ono (1978, 1982); and Thompson, 1989): (1) the low-cost firm, (2) the dominant firm (in terms of assets, employment, or share of the market, (3) a 'barometric' firm, and (4) the smallest firm. Dominant firm, barometric the smallest firm models are based upon empirical observations; no robust theoretical explanations exist to support why these types of market structures appeared. Ono argued that (at least from the theoretical point of view) the differences in cost structure are crucial in determining the relative positions of firms in oligopoly markets. A simple version of his theory of price leadership is as follows.

Consider Figure 2-5 which is assumed to represent the domestic FCOJ industry. The Florida marginal cost curve MC_f may be thought as one large firm's marginal cost or as a summation of a group of *'competitive fringe'* firms in a duopoly market. Suppose MC_b is Brazil's marginal cost. If Florida is a price leader, it faces demand curve FF'D' which is obtained by subtracting MC_b from DD' in the horizontal direction. Its corresponding marginal revenue is MR_f. Its optimal strategy is given at the point where $MC_f = MR_f$. Hence, its optimal price strategy is OE and its quantity strategy is EH and earns the surplus given by ACHE. Brazil, as the follower, gets the surplus JGE and sells EG of the market share. If Brazil behaves as the price leader, it faces demand BB'D', and it optimizes its profit at $MC_b = MR_b$. Therefore, its optimal price is OL and optimal quantity strategy is LK, with surplus JIKL. Florida as the follower sells LN with surplus ANL. Since, in this particular case, Brazil producer's surplus as the price leader (JIKL) is larger than that as the follower (JGE), Brazil will prefer the leadership position in the market. Florida producer's surplus as the follower (ANL) is larger than that as the leader (ACHE). Florida, therefore, prefers the follower position.This scenario may be represented in price-price space and is basically equivalent to Figure 2-3 with Brazil as

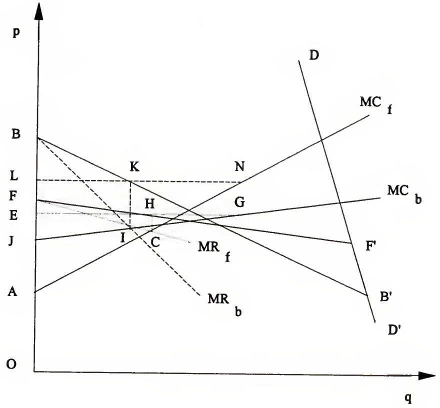


Figure 2-5 A price-leadership duopoly.

firm 1 and Florida as firm 2. The price leader-follower in this case naturally emerges due to the cost structure differences between the duopolists.

The theory of industrial economics/organization has made substantial progress in the past decade. The literature has grown rapidly in recent years, and game theory, which is the standard tool for the analysis of strategic conflict, has become prominent, and most of the traditional problems are reinterpreted in terms of this theory thus bringing a unified methodology to the field (Del Monte, 1992; and Tirole, 1988). Dynamics in industrial structure has come to replace static approaches which, for many real-world problems, are considered no longer appropriate. The next section briefly discusses the theory of dynamic games.

Dynamic Game Theory

Game theory is the analysis of rational behavior in situations involving interdependence of outcomes (Camerer, 1991) and is concerned with the mathematical study of conflict and cooperation (Bagchi, 1984). From the optimization point of view, game theory is concerned with more than single agent who seek to optimize differing objectives/criteria which may be in conflict with one another, and thus it can be viewed as an extension of standard optimization models. Many authors consider game theory such a powerful tool that it plays a central role in modern economic theory, and its use opens new frontiers to industrial organization and is used as a basic modeling tool in many other fields. In recent developments, attempts have been made to view 'games' as dynamic processes which lead to the development of the theory of dynamic games.

There have been numerous theoretical works of dynamic games developed in the forms of differential games. Early works in differential games (DG), optimal control theory (OCT), and dynamic programming (DP) appeared almost simultaneously, but independently of each other. *Pontryagin's Maximum Principle* in OCT and *Bellman's Principle of Optimality* in DP are basically the factors which led to the rapid development in the theories and applications of OCT and DP, respectively. The pioneering work of Isaacs (1967) in the theory of differential games originated from his research in military area, particularly related to warfare games of pursuit and evasion in 1954. Since then a considerable body of research in differential games has been developed. Even though the theoretical discussion on DG has been extensive, particularly in the last two decades, the real applications of DG as the analytical framework of research are still relatively limited.

Differential games (DG) are basically games in which the position of the players develops continuously in time (and usually are called difference games when formulated in discrete time). Loosely speaking, DG is a blend of (and the benefits from) game theory and optimal control theory. Some good introductory material on DG can be found in Kamien and Schwartz (1991), Başar and Olsder (1982), Başar (1986), and Case (1977). Kamien and Schwartz briefly discussed open-loop and feedback solution of Nash strategies in a differential game problem. The book by Başar and Olsder is probably one of the most comprehensive books in dynamic noncooperative game theory literature. Başar (1986) considered a general formulation of dynamic and differential games, which includes both discrete and continuous time problems as well as deterministic and stochastic. Case (1977) considered a Cournot problem to discuss the Nash-Cournot and Stackelberg solution and extended it to a dynamic game.

In game theory, the answer/solution to a problem may be obtained/found only if the concept of solution of the game is well-defined. Games that have the property that one player's gain is always another player's loss are called zero sum games. Games in which one player's gain does not necessarily mean another player's loss are called nonzero sum games. There are two basic theoretical approaches to problems of games. A game is called a *cooperative* game if the objective is to optimize the joint objective function (*e.g.*, the joint profits) of the group and then distribute them in such a way that no player is made worse off than if he or she did not cooperate with the other. Leitmann (1974) stated that the cooperative game is consistent with Pareto optimality, which is embodied in a statement 'I am willing to forego a gain if it is to be at your expense'. A game is called a *noncooperative* game if the players try to do the best for themselves without cooperation (typically, each player's interest is partly conflicting with others').

A typical two-player differential game may be written as player *i* choosing his control u_i, to maximize

$$
\begin{aligned}
&J^i = \int_{t_0}^{t_1} f^i(u_1(t),\ u_2(t),\ x_1(t),\ x_2(t),\ t)\ dt \\
&\mathit{subject\ \ to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2\text{-}1)\\
&\dot{x}_i(t) = g^i(u_1(t),\ u_2(t),\ x_1(t),\ x_2(t),\ t), \qquad\qquad i=1,2
\end{aligned}
$$

where $u_i(t)$ and $x_i(t)$, respectively, are the control/strategy and the state variables for player i in the system. The functions f^i and g^i are usually assumed to be known and continuously differentiable. Each player is aware of how the other player's choice of his control variable affects the state equations. Consequently, every player has to take into account the other player's decision in choosing his own. The Nash strategy assumes that the players choose their control variables simultaneously (and independently). When neither player has an incentive to revise his/her decision, the choices are said to be in equilibrium. The Stackelberg strategy assumes that one player may be in a position such that he can choose his control variable ahead of the other players. Dynamic duopoly games certainly are more difficult to solve than the static versions. Thus the exposition of dynamic games will begin from the following static analysis.

Following the terminology in Simaan and Cruz (1973a), a *static* duopoly game may be stated as follows. Let U_1 and U_2 be the sets of admissible strategies for players 1 and 2, respectively. Suppose that the profit functions $J^1(u_1,u_2)$ and $J^2(u_1,u_2)$ are two functions mapping $U_1 \times U_2$ into the real line such that Player 1 wishes to maximize J^1 and Player 2 wishes to maximize J^2.

Definition. If there exists a mapping T: $U_2 \rightarrow U_1$ such that, for any fixed $u_2 \in U_2$, $J^1(Tu_2,u_2) \geq J^1(u_1,u_2)$ for all $u_1 \in U_1$, and if there exists a $u_{2s2} \in U_2$ such that $J^2(Tu_{2s2},u_{2s2}) \geq J^2(Tu_2,u_2)$ for all $u_2 \in U_2$, then the pair $(u_{1s2},u_{2s2}) \in U_1 \times U_2$, where $u_{1s2} = Tu_{2s2}$, is called a Stackelberg strategy pair with Player 2 as leader and Player 1 as follower. So, the Stackelberg strategy is basically the optimal strategy for the leader when the follower reacts by playing optimally (see Siman and Cruz, 1973a).

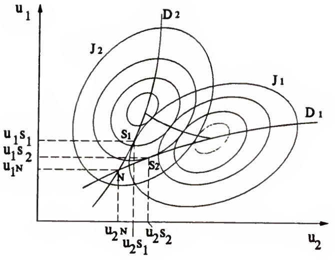
The graph $D_1 = \{(u_1,u_2) \in U_1 \times U_2 : u_1 = Tu_2\}$ of the mapping T is called the rational reaction set of Player 1. This set represents the collection of strategy pairs in $U_1 \times U_2$ according to which Player 1 reacts to every strategy $u_2 \in U_2$ that Player 2 may choose. By playing according to the set D_1, Player 1 is referred to as being a rational player. Let D_2 be the rational reaction set of Player 2 when Player 1 is the leader. The following propositions demonstrate the siginificant importance of the sets D_1 and D_2 in characterizing the Stackelberg and Nash strategies.

Proposition 1. A strategy pair (u_{1s2},u_{2s2}) is a Stackelberg strategy with Player 2 as leader iff $(u_{1s2},u_{2s2}) \in D_1$ and

Proposition 2. A strategy pair (u_{1N},u_{2N}) is a Nash strategy pair iff $(u_{1N},u_{2N}) \in D_1 \cap D_2$.

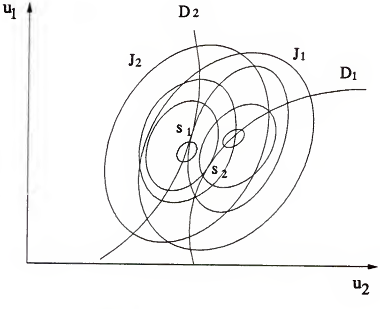
Following the above definition and propositions it can be stated that the leader in the Stackelberg solution achieves at least as good (possibly better) a profit function as the corresponding Nash solution. The reason is, by choosing a Stackelberg strategy, the leader actually imposes a solution which is favorable to himself. When the Stackelberg strategies with either player as leader coincide, then they both coincide with the Nash strategy (in this case the leader loses its advantage). In identical goal games, the Nash and Stackelberg strategies are the same.

As discussed by Simaan and Cruz (1973a), Nash and Stackelberg solutions may graphically be illustrated as shown by Figures (2-6) and (2-7). The figures are only abstractions of a timeless game in control variable space. It is essentially similar to Figure 2-2 (Cournot duopoly) but with nonlinear reaction curves. Note that in this case,



(Source: Simaan and Cruz, 1973a).

Figure 2-6 A duopoly game with Nash and Stackelberg solutions



(Source: Simaan and Cruz, 1973a).

Figure 2-7 A duopoly game without Nash but with Stackelberg solutions.

J^i is a cost function, so the objective is to minimize J^i, instead of maximizing it. S_1 (S_2) is the Stackelberg solution when player 1 (2) is the leader. N is the Nash solution (does not exist in Figure 2-7). Graphical illustrations demonstrate (see also Kamien and Schwartz, 1983) that when the reaction curves are linear, they only intersect once (a unique Nash solution) or they do not intersect at all (no Nash solution). Nonlinear reaction curves may result in either no intersection, one intersection (unique Nash) or multiple intersections (non-unique Nash equilibria).

At this point, it is probably worthwhile to provide a duopoly game with numerical examples. As illustrations for the game-theoretic approach to a duopolistic competition, the following examples of *payoff-matrix* games will be discussed.

Example 2-1. Consider the domestic FCOJ market. Suppose there are only two producers in the market, Florida and Brazil. Assume that each producer's desire is to maximize its own profit only. Let the profits of those producers be measured quantitatively by the entries in Table 2-1. The first entries correspond to the Florida producer's profit and the second entries correspond to the Brazil producer's profit. In this *payoff-matrix* game, let f_i be the strategy i of Florida ($i=1,2,3$), and b_j be the strategy j of Brazil ($j=1,2,3$). Thus, the Florida producer and the Brazilian producer simply want to maximize J^F and J^B, respectively.

If Brazil is the leader in the market, the rational reaction set of Florida is the set of pairs $D_F = \{(f_1, b_1), (f_3, b_2), (f_1, b_3)\} = \{(13, 12), (14, 14), (16, 13)\}$, and the Stackelberg strategy with Brazil as leader is the element of D_F that maximizes J^B, that is $(f_3, b_2) = (14, 14)$. In other words, by selecting strategy b_2, Brazil as the leader leaves no choice for Florida producer (the follower) but to choose strategy f_3. The Brazilian producer's profit with strategy b_2 is higher than what it would have been, had b_1 or b_3 been selected instead. If Florida is the leader, the rational reaction set of Brazil is D_B = $\{(f_1, b_3), (f_2, b_2), (f_3, b_3)\}$ = {(16, 13), (11, 13), (11, 17)}, and the Stackelberg strategy with Florida as the leader is (f_1, b_3) = (16, 13). The Nash strategy in this matrix game is $D_F \cap D_B = (f_1, b_3)$ = (16, 13). As we can see in this game, there is an incentive for Brazil to be the leader, since the Stackelberg strategy, when Brazil is the leader gives higher profit to Brazil than the Nash strategy. When Florida is the leader, however, the Stackelberg strategy is equal to the Nash strategy (Florida loses its advantage).

Table 2-1 The payoff matrix for Florida and Brazil.

	b_1	b_2	b_3	D_B
f_1	(13, 12)	(12, 10)	(16, 13)	(f_1,b_3) = (16, 13)
f_2	(10, 9)	(11, 13)	(12, 11)	(f_2,b_2) = (11, 13)
f_3	(11, 10)	(14, 14)	(11, 17)	(f_3,b_3) = (11, 17)
D_F	(f_1, b_1) = (13, 12)	(f_3, b_2) = (14, 14)	(f_1, b_3) = (16, 13)	

Example 2-2. Suppose now the game is extended to two stages. Let Table 2-2 represents the possible pairs of profits in the second stage of the game. It is assumed that the outcome of the first game does not affect the outcome of the second game. Similar to the first game, we can compute the Stackelberg and Nash strategies for this game. If Brazil is the leader, the rational reaction set of Florida is $D_F = \{(f_6, b_4), (f_5, b_5), (f_6, b_6)\}$ = {(16, 12), (13, 13), (14, 12)}, and the Stackelberg strategy is (f_5, b_5) = (13, 13). If Florida is the leader, the rational reaction set of Brazil is $D_B = \{(f_4, b_4), (f_5, b_5), (f_6, b_5)\} = \{(14, 13), (13, 13), (11, 14)\}$, and the Stackelberg strategy is $(f_4, b_4) = (14, 13)$. The Nash solution is $(f_5, b_5) = (13, 13)$. Contrary to the first game, the Stackelberg strategy when Brazil is the leader is equal to the Nash solution (Brazil loses its advantage). If Florida is the leader, however, the Stackelberg strategy is better than the Nash strategy.

Table 2-2 The payoff matrix in stage two.

	b_4	b_5	b_6	D_B
f_4	(14, 13)	(12, 8)	(13, 11)	$(f_4,b_4) = (14, 13)$
f_5	(15, 10)	(13, 13)	(12, 10)	$(f_5,b_5) = (13, 13)$
f_6	(16, 12)	(11, 14)	(14, 12)	$(f_6,b_5) = (11, 14)$
D_F	$(f_6, b_4) =$ (16, 12)	$(f_5, b_5) =$ (13, 13)	$(f_6, b_6) =$ (14, 12)	

Example 2-3. Suppose now the games are played consecutively. The profits to both players, as shown in Table 2-3, are the sum of profits obtained in Table 2-1 and Table 2-2. If Brazil is the leader, the rational reaction set of Florida is

$D_F = \{(f_1f_6, b_1b_4), (f_1f_5, b_1b_5), (f_1f_6, b_1b_6), (f_3f_6, b_2b_4), (f_3f_5, b_2B_5), (f_3f_6, b_2b_6), (f_1f_6, b_3b_4), (f_1f_5, b_3b_5), (f_1f_6, b_3b_6)\}$,

$= \{(29, 24), (26, 25), (27, 24), (30, 26), (27, 27), (28, 26), (32, 25), (29, 26), (30, 25)\}$, and the Stackelberg strategy is $(f_3f_5, b_2b_5) = (27, 27)$.

Table 2-3 The payoff matrix for the two-stage game.

	b_1b_4	b_1b_5	b_1b_6	b_2b_4	b_2b_5	b_2b_6	b_3b_4	b_3b_5	b_3b_6	D_B
f_1f_4	(27,25)	(25,20)	(26,23)	(26,23)	(24,18)	(25,21)	(30,26)	(28,21)	(29,24)	b_3b_4 = (30,26)
f_1f_5	(28,22)	(26,25)	(25,22)	(27,20)	(25,23)	(24,20)	(31,23)	(29,26)	(28,23)	b_3b_5 = (29,26)
f_1f_6	(29,24)	(24,26)	(27,24)	(28,22)	(23,24)	(26,22)	(32,25)	(27,27)	(30,25)	b_3b_5 = (27,27)
f_2f_4	(24,22)	(22,17)	(23,20)	(25,26)	(23,21)	(24,24)	(26,24)	(24,19)	(25,22)	b_2b_4 = (25,26)
f_2f_5	(25,19)	(23,22)	(22,19)	(26,23)	(24,26)	(23,23)	(27,21)	(25,24)	(24,21)	b_2b_5 = (24,26)
f_2f_6	(26,21)	(21,23)	(24,21)	(27,25)	(22,27)	(25,25)	(28,23)	(23,25)	(26,23)	b_2b_5 = (22,27)
f_3f_4	(25,23)	(23,18)	(24,21)	(28,27)	(26,22)	(27,25)	(25,30)	(23,25)	(24,28)	b_3b_6 = (24,28)
f_3f_5	(26,20)	(27,25)	(23,20)	(29,24)	(27,27)	(26,24)	(26,27)	(24,30)	(23,27)	b_3b_5 = (24,30)
f_3f_6	(27,22)	(27,25)	(25,22)	(30,26)	(25,28)	(28,26)	(27,29)	(22,31)	(25,29)	b_3b_5 = (22,31)
D_F	f_1f_6 = (29,24)	f_1f_5 = (27,25)	f_1f_6 = (27,24)	f_3f_6 = (30,26)	f_3f_5 = (27,27)	f_3f_6 = (28,26)	f_1f_6 = (32,25)	f_1f_5 = (29,26)	f_1f_6 = (30,25)	

If Florida is the leader, the rational reaction set of Brazil is

$D_B = \{(f_1f_4, b_3b_4), (f_1f_5, b_3b_5), (f_1f_6, b_3b_5), (f_2f_4, b_2b_4), (f_2f_5, b_2B_5), (f_2f_6, b_2b_5), (f_3f_4, b_3b_6), (f_3f_5, b_3b_5),(f_3f_6, b_3b_5)\}$,

$= \{(30, 26), (29, 26), 27, 27), (25, 26), (24, 26), (22, 27), (24, 28), (24, 30), (22, 31)\}$, and the Stackelberg strategy is $(f_1f_4, b_3b_4) = (30, 26)$. The Nash strategy is $(f_1f_5, b_3b_5) = (29, 26)$. Note that the Stackelberg (and Nash) strategies of the individual subgames are components of the Stackelberg (and Nash) strategies of the composite game. The nature of the solutions for this game is in the spirit of *Bellman's principle of optimality*. This example, however, is not truly dynamic in the sense that every player's position in second stage is not affected by its decision in the previous stage. This last example also demonstrates the possibility of *Stackelberg disequilibrium*. If both players know the outcomes, each of them will prefer the leadership position, since being a leader (if the other is the follower) will result in higher profits.

In most differential games literature, the strategies of selecting the control variables u_1 and u_2 are either *open-loop* or *feedback*. Open-loop strategies are ones where the players have no information on the actual state of the system. So, each player chooses all the values of his control variable for each point in time at the outset of the game; the control is the function of time, *i.e.*, $u_i = u_i(t)$. Feedback strategies are ones for which the players have perfect information about the actual state of the system. That is, the control at each point in time is a function of both time and the state of the system at that time, say $u_i = u_i(t, x_1(t), x_2(t))$, $i = 1, 2$. For more detail discussion on information structure in dynamic games, see Başar and Olsder (1982).

There are two types of equilibria that will be discussed here: open-loop and feedback equlibria. Let the nonzero-sum duopoly differential game problem of equation (2-1) be rewritten as

$$(i)\ J^i = max_{u_i}\left[\int_0^T e^{-\delta_i t} f^i(t,\ x_1,\ x_2,\ u_1,\ u_2)\ dt + e^{-\delta_i T} z^i(T,\ x_i(T))\right],$$
$$i = 1,\ 2$$

$$(ii)\ \dot{x}_i = g^i(t,\ x_1(t),\ x_2(t),\ u_1(t),\ u_2(t)), \qquad x_i(0) = x_0^i, \quad t \geq 0$$

$$(iii)\ u_i(t) = k^i(t,\ x_1(t),\ x_2(t)), \qquad u_i(.) \in R^+, \quad k : R^+ \rightarrow R^+, \qquad (2\text{-}2)$$

Here, we assume that each player has one type of control variable (u_i) and one type of state variable (x_i). Optimality conditions for (Nash) open-loop and feedback equilibria for this problem can be derived (see Starr and Ho, 1969 a and b; Başar and Olsder, 1982; and Kamien and Schwartz, 1991). The current value Hamiltonians are

$$H^i = f^i + \lambda_1^i g^1 + \lambda_2^i g^2, \qquad i = 1,\ 2 \qquad (2\text{-}3)$$

An open-loop solution assumes that the control variable *(u)* is a function of time only (not dependent on the states of the system). In other words, an open-loop solution assumes that each player commits to his/her entire sequence of actions through time at the outset of the game. By applying the standard necessary conditions of optimal control theory, the optimal open-loop trajectory must satisfy, the following conditions:

$$\begin{aligned}
&(i) \quad H^i_{u_i} = 0, && i = 1,\ 2,\\
&(ii) \quad \dot{\lambda}^i_j = \delta_i \lambda^i_j - \frac{\partial H^i}{\partial x_j}, && i = 1,\ 2; \quad j = 1,\ 2, \qquad (2\text{-}4)\\
&(iii) \quad \dot{x}_i = g^i(t,\ x_1,\ x_2,\ u_1,\ u_2), && x_i(0) = x^i_0, \quad i = 1,\ 2.
\end{aligned}$$

For the Nash feedback solution, the first (i) and the third (iii) equations of the necessary conditions (equation (2-4)) are the same. The second equation, however, can be written as

$$\begin{aligned}
&(ii\text{-}a) \quad \dot{\lambda}^1_j(t) = [\delta_1 \lambda^1_j - \frac{\partial H^1}{\partial x_j}] - [(\frac{\partial H^1}{\partial u_2})(\frac{\partial u_2^*}{\partial x_j})], \quad and\\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2\text{-}5)\\
&(ii\text{-}b) \quad \dot{\lambda}^2_j(t) = [\delta_2 \lambda^2_j - \frac{\partial H^2}{\partial x_j}] - [(\frac{\partial H^2}{\partial u_1})(\frac{\partial u_1^*}{\partial x_j})],\\
&\qquad\qquad\qquad\qquad j = 1,\ 2
\end{aligned}$$

By the first necessary condition, the term $(\partial H^i/\partial u_i)(\partial u_i^*/\partial x_i) = 0$. Hence, it does not appear in equation (2-5). The second term on the right hand side appears due to the assumption that the players employ feedback strategies and therefore the control u_j is a function of $(x_1(t),\ x_2(t))$. The presence of these last terms (*interaction terms*) makes feedback strategies difficult to compute. Due to the interdependences in the last terms and the fact that derivations usually end up with partial differential equations which are generally not easily solved, a different approach may be proposed. The most common approach to derive the feedback solution is by employing dynamic programming (DP).

The basic principle in DP is *Bellman's principle of optimality*, which can be stated as (Bellman and Dreyfus, 1962): *An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.* Each player's value function (the *Hamilton-Jacobi-Bellman* (HJB) equation, see for example, Başar and Olsder, 1982; and Kamien and Schwartz, 1991), is constructed and has the property :

$$- \frac{\partial J^i(t,\ x_1,\ x_2)}{\partial t} = max_{u_i}[e^{-\delta_i t} f^i(t,\ x_1,\ x_2\ u_P\ u_2) + \frac{\partial J^i(t,\ x_1,\ x_2)}{\partial x_1} + \frac{\partial J^i(t,\ x_1,\ x_2)}{\partial x_2}], \qquad i = 1,\ 2 \tag{2-6}$$

Some duopoly game models can be found in Case (1971), Simaan and Takayama (1978), Feichtinger and Dockner (1985), Driskill and McCafferty (1989), and Karp and Perloff (1993). Case's work (1971) is among early references. His paper discusses a problem of profit maximization for two firms manufacturing the same commodity that is cast as a differential game. Simaan and Takayama (1978) consider a dynamic duopoly where two firms, each having an upper limit on the production rate and sharing the same market, maximize their profits over a fixed planning period. The model assumed quadratic production costs and a linear demand function.

Murphy *et al.* (1989) provided an analysis of a Nash dynamic game model for investigating public and private sector oil inventory policies in unstable world oil markets. The model assumed that the private player was risk-neutral and maximized the expected present value of arbitrage profits plus net convenience benefit (the operational value of inventories in smoothing refinery processes and avoiding transportation bottlenecks minus holding costs), acts competitively, treating market prices for oil as independent of his/her own inventory decisions. The public player (government) was also assumed to be risk neutral and maximized the expected present value of consumer surplus derived from oil imports, plus the net convenience benefit of private inventory, minus the holding cost of the government stock.

Feichtinger and Dockner (1985) provides a study of a differential games model of a duopoly profit-maximizing firms competing for the same stock of customers. The study emphasized the solution technique, rather than the economic significance of the proposed model. The model assumed that the customers of each firm are driven away gradually by increasing product prices. Since the state variable is absent from the Hamiltonian maximizing conditions as well as from the adjoint equations, open-loop Nash solutions can be obtained.

Driskill and McCafferty (1989) developed a differential-game model of duopoly in which firms incur costs associated with how fast they change their level of output. Their analysis solved a subgame perfect equilibrium to the model in which the steady-state level of output lies strictly between the static Cournot and perfect competition solutions. They also demonstrated that the limit game in which the speed of adjustment parameter approaches zero does not tend toward the static Cournot equilibrium.

The work by Karp and McCalla (1983) is probably among early references in dynamic game theory applied to an agricultural trade problem (world corn market). They considered buyers and sellers as the players in the game and sought a Nash noncooperative equilibrium as the solution to their dynamic difference game. They explored numerical results through simulation under various scenarios in trade policies.

More recently, Karp and Perloff (1993) did empirical work by developing a linear-quadratic dynamic feedback oligopoly model to estimate the degree of competitiveness and the adjustment paths of the two largest coffee exporters, Brazil and Colombia. They developed a flexible model to allow for the possibility that firm 1 (Brazil) and firm 2 (Colombia) act like price takers, collude, or export oligopolistic levels between those two extremes.

Previous Studies on Domestic Orange Juice Industry

Among previous studies related to Florida orange industry are by Behr (1981), Irias (1981), and McClain (1989). Behr (1981) developed an econometric model to estimate the parameters of the mathematical futures trading model using cross-section and time series data. He found that industrial concentration has a quadratic effect on hedging activity, which suggests the causal linkage between futures market and market structure. He concluded that futures markets evolve in response to the needs of hedgers, which in turn are shaped by the dynamic forces of the economic conditions that beset commercial interests. Irias (1981) devised an econometric model to study international trade of frozen concentrated orange juice. A system of structural-form equations was estimated by two-stage least squares. He found that Florida's wholesale prices, orange production forecasts, US import and export prices, exchange rates and the occurence of a freeze in Florida played major explanatory roles in the various price equations. He also found that US imports of FCOJ from Brazil had an elastic response to the price differential (Florida's wholesale price minus import price).

Using Monte Carlo simulation techniques, McClain (1989) built a dynamic-stochastic model of the world orange juice market and projected the long-run average levels of prices, production, and trade. Supply is projected based on the age distribution of trees and yields by age class, while demand is describe as a function of the world price, expected growth rate, and regional tarriffs. She also incorporated the effects of weather through the random selection of tree death and yield rates from historically defined distributions.

Studies by Malick (1980), Powe (1973), Powe and Langham (1980), and Mataya (1983) are related to production-inventory issues. Malick estimated a simultaneous equation model of the Florida retail orange juice marketing system. He found that the roles of processors' inventory and expected future supplies in determining price are significant, reflecting the importance of freezes and threat of freezes. The consumer demand for FCOJ is slightly inelastic, and the consumers tend to increase stocks of FCOJ when price is relatively low.

Powe (1973), and Powe and Langham (1980) developed a dynamic model of the Florida orange industry to simulate the effects on industry performance of alternative inventory, pricing, advertising, and supply control policies. Their model was composed of ten interrelated sectors, extending from tree planting through consumer demand. They found that policies that reduced long-run supplies of orange products caused substantially higher aggregate grower profits, lower storage costs, and higher retail prices. They also reduced risks for orange producers, but not for handlers and consumers. An interesting characteristic they observed in the policy analysis was the presence of conflict of interest among growers, handlers, and consumers. In order for one group of participants to gain, another was placed in a less desirable position.

Mataya (1983) developed a storage model for Florida FCOJ centered on expectations about the future market conditions. An econometric model was used to analyze annual FCOJ data (1967-1982). His study indicated that among the exogenous variables, the futures price or the proxy for price expectation has the greatest influence on the behavior of the storage model. It causes a positive response in ending inventories to the future price. As suggested by the storage theory of Helmberger *et al.* (1982), the futures price is an allocative mechanism between consumption and storage. Moreover, he found that an increase in the beginning inventory has a positive effect on the expected carry-overs, but expected market clearing price and expected revenue increase at a decreasing rate. The consequence of this is that the market will channel FCOJ into storage as long as the price for storage increases, but consumers' marginal utility for storage starts declining when there is an excess inventory level. Therefore, price has to go down. The model showed that among the parameters, the demand and supply intercepts (*i.e.*, growth in demand and supply) had the most substantial impact on the model. A shift in the demand curve resulted in a decline in the expected carry-overs. The opposite result occurred when there was a shift in supply, in which storage of excess supply prevents a dampening effect on prices.

Of the various references related to the citrus industry, the most important one is undoubtedly the work of Ward and Kilmer (1989). The text, as the title says, provides a domestic and international perspective about the industry. Thus, it provides a broad relevant aspects to this dissertation, particularly in the process of understanding and analyzing the subject area, and for conceptualizing the theoretical models.

CHAPTER 3
THEORETICAL MODELS OF THE WORLD FCOJ MARKET

As has already been briefly discussed in the previous chapter, the structure-conduct-performance (SCP) paradigm is the most established approach in the industrial organization/economics (IO) literature. Among the criticisms frequently addressed to the traditional form of SCP are (see for examples in Reid, 1987; Tirole, 1988; and Jacquemin, 1987): it is too *rigid* (in terms of the way in which causality is assumed to operate, and of the factors which are usually considered within each of the three category used) for robust theoretical development; and it places too much emphasis on empirical studies developed upon (usually) *loose* theories. In contrast to the attacks by the opponents of this approach, with carefull interpretation and adoption, SCP potentially provides a rich and flexible framework for market analysis to help understand the complexity of reality. Economic theories relevant to IO develop through time. Yet they still fit the framework with different emphasis on the four important elements of the approach (the basic conditions, structure, conduct, and performance).

In the early development of IO literature, more emphasis was given to empirical studies. While more recently, theoretical works seem to be increasingly popular. The case study in Mason's work (see Reid, 1987) was probably the starting point for generalization to SCP. Post-Mason development such as Bain, stressed the formulation of direct empirical links between market structure and economic performance, deemphasizing intermediate conduct (Scherer, 1980). Scherer, in contrast, concentrated mainly on business conduct spanning structure and performance. Shubik and Levitan (1980), who attempted to discuss market structure and behavior, pointed out that the methodology of game theory provides a new and useful approach to studying the interrelationship between structure and behavior. Similarly, other theoretical approaches to IO such as Jacquemin (1987), Friedman (1983), Tirole (1988) still fit the SCP framework, with different emphasis on its elements. Thus, a possible important path is the development of more rigorous theoretical foundations of imperfect competion and to test them empirically, even though theoretical and empirical works do not necessarily take place in the same research project at the same time. In this respect, two steps will be executed in this study: First, theoretical models of the system under study will be developed and investigated to provide foundations for the second step, the empirical analysis.

A considerable amount of effort has been devoted to developing microeconomic theories with the aid of optimization/quantitative methods aimed at analyzing the *economic efficiency*, which consists of *technical efficiency* and *allocative efficiency*. For many results in microeconomic theory, economic actions are treated as static either under monopoly or perfect competition assumptions. Too often in the real world, however, market competitions fall somewhere in between, and the time dimension is critical so that the theory would be of limited usefulness if it did not apply to an intertemporal/dynamic decision making environment.

In the previous chapter, some competing theories of imperfect competition were discussed. The conjectural variation (CV) concept seems appealing for at least two reasons. First, theoretically it offers a systematic approach and allows flexibility in interpretation to capture the interactions (*i.e.*, with respect to market power/degree of competitiveness, behavioral assumptions and/or solution concepts) between economic agents under study. Second, it provides an *operational* measure for testing particular behavioral assumptions about players' behavior or for estimating the degree of market power in empirical studies.

Most of the empirical studies previously discussed were developed within static frameworks. Friedman (1983, p.110) criticizes conjectural variation in static models for three reasons: (1) Models are not actually dynamic (that is, time makes no explicit appearance); (2) Firms are assumed to maximize current-period profits, rather than discounted profits over some horizon of time; (3) Firms have expectations about how their rivals will behave that need not be correct (over time, they will receive information, and even if it contradicts their expectations, they will go on believing as before). Some empirical studies discussed earlier also indicate unsatisfactory results of static conjectural variation models.

The foregoing discussion provides the basis of the need to view the competition in imperfect markets as a dynamic process rather than a static one. Many of decisions in imperfect markets can be described as parts of strategic interactions among players.Thus, game-theoretic models appears to be promising analytical tools. With higher growth in supply than growth in demand of FCOJ (for example see FDOC, 1993b; and World Bank, 1993), it may be expected that the battle for markets will increase in intensity in the coming years. The model proposed in this study departs from the premise that Florida and Brazil orange juice producers, as rational economic decision-making agents in the system under study, inevitably have to discover how to manage their strategies for their survival and benefits over time. The interdependence and competitive interaction among players/participants is a key feature of imperfect competitions. Hence, trying to anticipate the competitive response of rival players is an exercise that no player can afford to neglect.

Five interrelated factors in this study will be considered: (1) the time framework of the analysis; (2) the objective(s) of the players/decision makers (DM); (3) the buyers' willingness to pay for the product(s) (demand side); (4) the cost of producing and marketing product (supply side); and (5) the setting of the competition (*structure* and *conduct*).

The proposed models deal with a (relatively) short-run planning horizon (one year). In this study, producers, players or decision makers actually refer to processors of FCOJ, and not growers. Although institutional arrangements in the citrus industry are undoubtedly complex, for the practical purposes in this study, drawing industry boundaries is unavoidably a matter of judgement. As the modeling effort is impossible without any simplification, some assumptions have to be made in the study. First, since we are interested in analyzing the role of the main players in the world OJ market *(i.e.,* Florida and Brazil), and both players' supplies account for the most of the market share, it is assumed that the system under study can be considered as a duopoly market. This implies that buyers are assumed to be homogeneous, infinitely large in number, and (even if not too large) do not have or do not exercise market power.

It can be argued that each player will prefer higher returns to lower returns, *ceteris paribus*. Moreover, it is plausable to assume that they make the best use of information they can. The proposed models examine the storage behavior of these two players based on simplified present market structures. Unlike the study by Mataya (1983) where an econometric model is developed based on the Helmberger-Weaver-Haygood storage model, a different approach is taken in this study. A dynamic duopoly game model is developed to simulate the participants' optimum behavior patterns.

Roughly, there are three major FCOJ markets (Figure 3-1) in the world: the U.S., European markets, and the rest of the world (ROW). Although production and consumption in the rest of the world (ROW) may be increasingly important, in this short-term model they will be assumed to not be determining factors in the model. Assuming that both participants are profit maximizers, each has to solve a dynamic program to optimize inventory decisions given a set of constraints and the other player's policy. In this model, the players essentially have to make intertemporal and spatial decisions in this duopolistic competition. The setting leads to a dynamic game as one of the most natural approaches to the problem. In a dynamic game, however, the concept of solution is not well-defined unless further assumptions are made. Clearly Florida competes with Brazil, and so it is assumed that there is no cooperation between the two players (that is, this is a noncooperative competition).

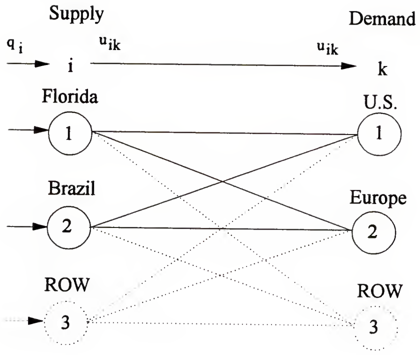


Figure 3-1 The schematic of the world FCOJ market under study.

Brazil is the world's largest producer/exporter of FCOJ. On the other hand, the U.S. is still the largest FCOJ market in the world, and Florida producers have the largest share of the domestic FCOJ market. Consequently, both Brazil and Florida producers play important roles in FCOJ pricing in both the world and domestic orange juice markets.

It may always be arguable whether profit maximization is the most appropriate objective of firms. Yet most economic analyses assume the objective of profit maximization. This is not merely out of habit. For one reason, profit maximization may bring about other firm's objectives automatically. In addition, the profit-maximization assumption may provide predictions about firm behavior as well as or better than any alternative paradigms. The profit-maximizing assumption is a useful aid in obtaining a general understanding and explanation of the behavior of groups of firms (Stigler, 1952). Friedman (1953) argued that the profit motive is the strongest, the most universal, and the most persistent of the forces governing business behavior. Thompson (1989) propounded that, in general, firms confronted with strong competitive pressures are prone to exhibit short-run profit-maximizing behavior. He summarizes that the assumption of profit-maximizing behavior is especially suitable in those situations where (1) large groups of firms are involved and nothing has to be predicted about the behavior of individual firms; (2) competitive forces are relatively intense; (3) the general effects of a specified change in conditions upon prices, output, and resource inputs are to be explained and predicted rather than the values of these magnitudes before or after the change; and (4) only the directions of change are sought rather than precise numerical results.

From the theoretical point of view, it has been recognized that the choice of a strategic/decision variable is critical to the solution of imperfect competions. Unfortunately, there are no hard-and-fast rules to resolve the matter clearly. Perhaps, an appropriate consideration is what strategy/decision sets are available and relevant to the players/decision makers. DMs have relatively fewer strategy/decision making instruments available to compete in a market in a relatively short-run period than those in the long run. Thus, as a gross simplification, the speed at which they can alter these instruments is the main consideration in conceptualizing the strategic/decision variables in the dynamic duopoly models developed here. Other important considerations in determining these instruments are the nature of the product, the production technology, and the feature of the market.

Among the important strategies popularly studied in imperfect competitions is advertising. An effort to develop effective advertising strategies was studied in Hochman *et al.* (1974). In their model, a sales response to advertising is estimated. Then, an optimal control model with advertising expenditure as the control is developed. The results show gains that can be realized through inter- and intraseasonal variations in the level of advertising expenditures. Thus, optimal path of adjustment in both advertising and sales over time is established that provides indications for an effective advertising policy.

Lee and Fairchild (1988) reported that demand expansion through advertising of Florida orange juice has resulted in increased orange juice imports through non-Florida ports as well as increased sales from Florida. Interestingly, they found that non-Florida imports are more responsive than Florida sales to Florida orange juice advertising. They concluded that, on a volume basis, free riders appear to benefit more from the Florida orange juice advertising program than do Florida orange growers.

Due to the long productive life of orange trees, the production of FCOJ in the short-run may be assumed to be fixed (planting decisions are made several years prior to harvest which implies that total production in any particular season is predetermined). In other words, supplies are treated as exogenous to the system. A rule of thumb method (see Rosenbaum, 1975) for estimating the production of Florida FCOJ for a given pack year in progress is to multiply the product of the latest USDA Florida orange production estimate for that season, the proportion of the crop utilized for concentrate, and the most recent USDA projected juice yield factor. Seasonality in orange (and so FCOJ) production results in the appearance and disappearance of stocks in a particular year. Thus marketing and inventory decisions, rather than production decisions, will be decisive factors in short-run planning. Pricing is one of the most important elements of the marketing decision variables as it is the variable that directly determines the revenue.

In this study, two theoretical models are considered: (1) the quantity (Cournot) game; and (2) the price (Bertrand) game. In the quantity game, it is assumed that both DMs select the quantity to be sold in maximizing their profits. It may also be interpreted that DMs set their product prices by choosing their profit-maximizing quantities.

Similarly, in the price game, DMs determine their product prices to maximize their profit.

To mathematically state the models, each player maximizes the net present value of its profit over a finite time horizon (one year) subject to its inventory constraints:

$$Maximize_{d_{ik}(t)} J^{(i)} = \int_{t_0}^{T} (e^{-\delta_i t} \sum_{k=1}^{3} \Pi_k^{(i)}(d_{ik}(t),\ I_i(t),\ t))\ dt + e^{-\delta_i T} S_i I_i(T), \tag{3-1}$$

where d_{ik} is the strategic/decision variable chosen by player i with respect to market k (that is, $d_{ik}(t) = u_{ik}(t)$ in the quantity game model, and $d_{ik}(t) = p_{ik}(t)$ in the price game model). The inventory level for player i at time t is $I_i(t)$ with salvage value S_i, and discount rate δ_i. Each model assumes that the players impose the terminal-point condition $(S_i\ I_i(T))$, in which $I_i(T)$ may represent the present common practice in the processed citrus industry to try to maintain a sufficient year-ending inventory level. Assuming that the selling costs for player i to market k are $C_{ik}(u_{ik})$ and its inventory cost is $G_i(I_i)$, equations (3-1) may be written as (for simplicity, t argument is suppressed)

$$\begin{aligned} Maximize_{d_{ik}} J^{(i)} = \int_{t_0}^{T} (e^{-\delta_i t} \sum_{k=1}^{3} (p_{ik}(t)\ u_{ik}(t) - C_{ik}(u_{ik}(t)) - G_i(I_i(t))\ dt \\ + e^{-\delta_i T} S_i I_i(T). \end{aligned} \tag{3-2}$$

S_i in equation (3-2) is the salvage value of the inventory (I_i) at the final period T. The optimization problem is constrained by the accounting identity relating inventory change to sales and production (*the equation of motion*) and can be written as

$$\dot{I}_i(t) = q_i(t) - \sum_{k=1}^{3} u_{ik}(t). \tag{3-3}$$

In this case, $q_i(t)$ refers to the i-th player's production level at time t (assumed to be fixed). Each producer sells the amount of $u_{ik}(t)$ at time t to the market k and charges the price $p_{ik}(t)$. When production exceeds sales then the difference is added to the inventory. Otherwise, excess demand is taken from the inventory.

In these dynamic duopoly game models, there are at least five crucial elements to be considered: (1) the assumption on the functional form(s) of (inverse) demand; (2) the assumption on the cost function(s) (processing-, selling- and inventory-related costs); (3) the choice of the strategic/decision variable; (4) the behavioral assumption of each player about the other player; and (5) the inventory policy.

Demand function(s) in the price game or inverse demand function(s) in the quantity game, together with cost functions determine the concavity of the objective functions (profits) to be maximized. As is known from optimal control theory (OPT), *Pontryagin's maximum principle* provides a set of necessary conditions, but in general, these conditions are not sufficient. When certain concavity conditions are satisfied, however, as the *Mangasarian's sufficiency theorem* says that the conditions determined by the maximum principle are sufficient for maximization (see *e.g.*, Mangasarian, 1966; Kamien and Schwartz, 1991; and Chiang, 1992).

It has been recognized that in imperfect competition, the equilibrium attained depends on the choice of strategic/decision variable and the assumption regarding the rival's response (*i.e.*, the conjectural variations). It can be stated also that functional assumptions and behavioral assumptions in an imperfect competition model will stipulate the reaction functions of competing players. In addition, in the dynamic models developed here, as will be shown in the next section, the underlying assumption related to the inventory policy is critical to model solution.

It is generally argued that the behavior of buyers determines the demand forces in a market. Economists have long attempted to develop testable hypotheses about consumer behavior. As suggested by economic theory, demand for a product depends on various factors such as product price, its attributes, consumer tastes and preferences, consumer income, prices of related items, consumer expectations, etc. In actual practice, however, there is no easily identifiable demand function from which elasticities can be calculated. Observed prices and quantities are the result of a dynamic, everchanging system (Dahl and Hammond, 1977). Adding to the obscurity, Tomek and Myers (1993) are probably right in pointing out that evidence is growing that published empirical results cannot be confirmed.

Difficulties in specifying an appropriate demand function for FCOJ are due to various sources. For example, utilization of citrus includes various products (such as fresh, frozen concentrate (FCOJ), chilled juice (COJ), canned juice, and blends), which are likely related to each other. In the U.S., the fact that imported FCOJ is used in conjunction with domestic FCOJ to produce FCOJ and COJ and some imports enter through non-Florida ports, implies that to some extent, they are probably gross substitutes and gross complements at the same time. Also, distribution channels for FCOJ (bulk, retail, and institutional) may play important roles in determining the *shape* of demand for FCOJ. The effort to specify an *appropriate* functional form for demand is eventually confronted with the classical dilemma of the *bargaining game* of discovering the balance between mathematical (and statistical) convenience/simplicity and realism.

Since there are only two types of decision-making instruments that are considered here, this study is particularly interested only in the effects of these instruments in the demand function (or the market response to these instruments). To allow analytical solution of the dynamic models, admittedly simple demand models need to be assumed. Thus, a *partial* demand specification, rather than a structural demand-system specification, is adopted here. Furthermore, it is assumed that, in this short-run framework, a seasonal factor may play an important role. In the quantity game, in which players use quantity as their strategic instruments, their individual inverse demand functions are assumed to be

$$\begin{aligned} p_{ik}(t) = P_{ik}(u_{ik}(t),\ u_{jk}(t),\ \bar{s}_{ik}(t)), \qquad & i, j = F(lorida),\ B(razil), \\ for \quad k = U(S),\ E(urope); \qquad & and \quad i \neq j, \end{aligned} \tag{3-4}$$

where p_{ik} is the price of the product from producer i in the market k at period t, and $\bar{s}_{ik}$ is a vector of exogenous variables. It is assumed that the inverse demand functions are negatively sloped ($\partial p_{ik}/\partial u_{ik}<0$, holding the rival's quantity constant). It is generally expected that the products may be either gross substitutes (when $\partial p_{ik}/\partial u_{jk}<0$) or gross complements (when $\partial p_{ik}/\partial u_{jk}>0$), holding its own quantity constant.

In the price game, the players are assumed to use price as their strategic instruments, and the associated individual demand functions are

$$\begin{aligned} u_{ik}(t) = U_{ik}(p_{ik}(t),\ p_{jk}(t),\ \bar{a}_{ik}(t)), \qquad i, j = F(lorida),\ B(razil), \\ for \quad k = U(S),\ E(urope), \qquad and \qquad i \neq j, \end{aligned} \tag{3-5}$$

and they are also negatively sloped ($\partial u_{ik}/\partial p_{ik}<0$), indicate substitutes (when $\partial u_{ik}/\partial p_{jk}>0$) and complements (when $\partial u_{ik}/\partial p_{jk}<0$), and $\bar{a}_{ik}$ is also a vector of exogenous variables.

Furthermore, it is assumed that the cost of producing, processing, and selling FCOJ (for producer i to market k) is a function of the quantity sold, $u_{ik}(t)$ and *market-separable*, which means that the selling cost to each market is independent of each other:

$$C_{ik}(t) = C_{ik}(u_{ik}(t)) \tag{3-7}$$

The cost of inventory for producer i is a function of the level of inventory held at period t, $I_i(t)$:

$$G_i(t) = G_i(I_i(t)) \tag{3-8}$$

CHAPTER 4
CHARACTERIZATION OF THE OPTIMAL SOLUTIONS: A THEORETICAL ANALYSIS

For duopoly/oligopoly models in general, the static to dynamic, dynamic open-loop to dynamic feedback, and non-spatial to spatial solutions are progressively complex. The analysis in this section will start with the derivation of non-spatial dynamic open-loop solutions. In this chapter, general solutions to two models of games: the dynamic duopoly quantity game (DDQG) and the dynamic duopoly price game (DDPG) are examined.

First, it is assumed that the costs of producing and selling for both players are *market specific* and *independent* of each other (equation (3-7)). These costs can be approximated by linear cost curves (c_{ik} are constants) and take the form

$$\begin{aligned} C_{ik}(u_{ik}(t)) &= c_{ik}u_{ik}(t), \qquad for \quad k = U(S),\ E(urope), \\ & \qquad where \quad i = F(lorida),\ B(razil). \end{aligned} \tag{4-1}$$

Inventory related costs also have linear form (h_i are constants)

$$G_i(I_i(t)) = h_i I_i(t), \qquad i = F(lorida),\ B(razil). \tag{4-2}$$

The Dynamic Duopoly Quantity Game (DDQG) Model

Nonspatial Open-loop Solutions to the DDQG Model

As the starting point, it is assumed that there is only one market for competition with two players. The generalization to multimarket/spatial model is, in principle, straightforward. In this section, the open-loop solution for a one market competition (nonspatial, that is, *k* in equations (3-2) is equal to 1) model is derived. The current value Hamiltonians are (*t* argument is suppressed):

$$H_i = \underset{[1]}{[p_i u_i - C_i - G_i]} + \underset{[2]}{[\lambda_i^i (q_i - u_i)]} + \underset{[3],}{[\lambda_j^i (q_j - u_j)]} \qquad (4\text{-}3)$$

$$where \qquad i = F(lorida),\ B(razil)$$

Note that λ_i^i and λ_j^i are the current value *own-costate/adjoint* and *cross-costate/adjoint variable* with respect to the rate of change of inventory.

The first term ([1]) in the current value Hamiltonians (4-3) essentially represents the *current-profit effect* of u_i, while the second and the third terms ([2] and [3]) reflect the *future-profit* effects of u_i, since the purpose of inventory policy is for future sales/revenues. The third term ([3]) may require further interpretation (it will be discussed further later) as it is related to the rival's inventory policy. By nature, the two profit effects are competing, since a dynamic/intertemporal optimization basically deals with decisions/policies about present *versus* future time. The model delineates the overall profit prospect of various policies for each player, with both immediate and future effects, together with its rival's reactions taken into account.[1]

Assume that u_i is unconstrained, so that the optimal u_i is an interior solution. If it obeys the corresponding equation of motion, then the first order necessary/optimality conditions are

$$\frac{\partial H^i}{\partial u_i} = H_{u_i} = p_i + u_i\frac{\partial p_i}{\partial u_i} + u_i\left(\frac{\partial p_i}{\partial u_j}\frac{\partial u_j}{\partial u_i}\right) - \frac{\partial C_i}{\partial u_i} - \lambda_i^i - \lambda_j^i\left(\frac{\partial u_j}{\partial u_i}\right) = 0, \qquad (4\text{-}4a)$$

which can be rewritten as

$$p_i + p_i\left(\frac{\partial p_i}{\partial u_i}\frac{u_i}{p_i}\right) + p_i\left(\frac{\partial p_i}{\partial u_j}\frac{u_j}{p_i}\right)\left(\frac{\partial u_j}{\partial u_i}\right)\left(\frac{u_i}{u_j}\right) - MC_i - \lambda_i^i - \lambda_j^i\,\gamma_{ij} = 0. \qquad (4\text{-}4b)$$

Rearranging (4-4b) gives

$$p_i\left(1 + \frac{1}{\eta_{ii}} + \frac{\gamma_{ij}}{\eta_{ij}}\frac{u_i}{u_j}\right) - MC_i - \lambda_i^i - \lambda_j^i\,\gamma_{ij} = 0, \quad i \neq j \qquad (4\text{-}4c)$$

---

[1] In principle, from an analyst's point of view, the model may be considered as a form of a dynamic multiobjective (in this case, two objective) optimization problem.

where MC_i, $1/\eta_{ii}$, $1/\eta_{ij}$, and γ_{ij} are, respectively, the marginal cost of *processing* and *selling*, the own-price flexibility of demand (the ratio of the percentage change in its own price to the percentage change in its own quantity, holding the rival's quantity constant), the cross-price flexibility of demand (the ratio of the percentage change in its own price to the percentage change in the rival's quantity, holding its own quantity constant), and the conjectural variation (CV) in-quantity of player i about player j *(i.e.*, $\partial u_j/\partial u_i$, which is assumed to be constant throughout the planning period). The optimality conditions (4-4c) may be rearranged to give

$$p_i = \frac{MC_i + \lambda_i^i + \lambda_j^i \gamma_{ij}}{(1 + \frac{1}{\eta_{ii}} + \frac{\gamma_{ij}}{\eta_{ij}} \frac{u_i}{u_j})}. \tag{4-5}$$

Provided that an inverse demand function (p_i and p_j) can be specified, a closed-form solution to (4-5) (for u_i and u_j) can be obtained. The current value own-costate variable, λ_i^i, measures the value (with respect to player i) of an additional unit of its own stock of inventory, and the current value cross-costate variable, λ_j^i, measures the value (with respect to player i) of an additional unit of the other player's stock of inventory. Suppose that the conditions

$$\Lambda_{ij} = \begin{cases} 1, & i = j, \\ a_{ij}, & otherwise \end{cases} \tag{4-6}$$

represent the *valuation* by player *i* about its rival's end-of-year inventory policy. For example, the presence of end-of-year inventory in Brazil is not only valuable for Brazilian producers but also for Florida, then a_{ij} is not zero, rather it may take on a value between 0 and 1.

Formally, the transversality conditions are given by

$$I_i(T) \geq 0, \quad \lambda_j^i(T) \geq S_i \Lambda_{ij} = \begin{cases} S_i, & i = j, \\ S_i a_{ij}, & otherwise, \end{cases}$$
$$and \tag{4-7a}$$
$$I_i(T)(\lambda_i^i(T) - S_i \Lambda_{ij}) = 0.$$

To simplify, it may be assumed that $(I_i(T), I_j(T))$ are always > 0, or they are free, so (4-7a) reduces to

$$\lambda_j^i(T) = S_i \Lambda_{ij} = \begin{cases} S_i, & i = j, \\ S_i a_{ij}, & otherwise. \end{cases} \tag{4-7b}$$

The costate or adjoint equations:

$$\begin{aligned} &(i)\ \dot{\lambda}_i^i = \delta_i \lambda_i^i - \frac{\partial G_i}{\partial I_i}, \\ &(ii)\ \dot{\lambda}_j^i = \delta_i \lambda_j^i - \frac{\partial G_i}{\partial I_j}, \qquad i,\ j = F(lorida),\ B(razil) \end{aligned} \tag{4-8}$$

The second term on the right hand side of (4-8 *(ii)*) is zero, however, since by assumption the cost of inventory of a player is independent of the rival's inventory decisions. Equations (4-8) together with the transversality condition of equation (4-7b) yield

$$\begin{aligned} &(i)\ \lambda_i^i(t) = \frac{1}{\delta_i}\left(MG_i + (\delta_i S_i - MG_i)\, e^{\delta_i(t - T)}\right), && \textit{for} \quad i = j, \\ &(ii)\ \lambda_j^i(t) = a_{ij} S_i e^{\delta_i(t - T)}, && \textit{for} \quad i \neq j \end{aligned} \tag{4-9}$$

where $MG_i = (\partial G_i/\partial I_i)$ is the marginal cost of inventory (which in this case are the constants h_i). Note that for this model, when the marginal cost of inventory is constant, the Hamiltonians (4-3) are linear in the state variables, and the optimality conditions (4-4c) or (4-5) are free of the state variables. Similarly, the state variables also disappear from the adjoint/costate equations (4-9). These properties are important for analyzing the feedback solutions. Equation (4-9) indicates that as discount rate becomes larger, *ceteris paribus*, the costate becomes less valuable and hence the optimal inventory is lower.

The state equations are

$$\dot{I}_i = q_i - u_i \tag{4-10}$$

Assuming that the Hamiltonians H^i are jointly concave in u_i and I_i, and since the equations of motion are linear in u_i and in I_i, the necessary conditions (4-4c) - (4-10), by Mangasarian theorem are also *sufficient* for the optimality of the solution for each player.

Nonspatial Feedback Solutions to the DDOG Model

The feedback dynamic game solutions, in general, are difficult to derive. As discussed in Chapter 2, nonspatial feedback solutions are different from the nonspatial open-loop solutions only in the costate/adjoint conditions of (4-8). For the feedback solutions, the costate equations (derived from (2-5)) are

$$(i)\ \dot{\lambda}_i^i = \delta_i \lambda_i^i - \frac{\partial G_i}{\partial I_i} - \left(\frac{\partial H^i}{\partial u_j}\right)\left(\frac{\partial u_j^*}{\partial I_i}\right),$$

$$(ii)\ \dot{\lambda}_j^i = \delta_i \lambda_j^i - \left(\frac{\partial H^i}{\partial u_j}\right)\left(\frac{\partial u_j^*}{\partial I_j}\right), \qquad i, j = F(lorida), B(razil). \tag{4-11}$$

Due to the special structure of the model, a simpler approach may be taken by examining the properties of the Hamiltonians (4-4c) and the costate/adjoint equations (4-11). By definition, the controls in feedback solutions depend on the state variables. Thus, if it turns out that the first optimality conditions result in the *state separability* (*i.e.*, independent of the state variables), and the equations of motion are linear in the state variables (hence they disappear from the costate equations), then it may be concluded that the feedback solutions coincide with the open-loop solutions. The definition similar to that by Dockner *et al.* (1985) (see also Feichtinger and Dockner, 1985) is utilized.

Definition . A differential game (DG) is *state separable* if

$$(i)\quad H^{i}_{u_i I_j}\Big|_{H^{i}_{u_i}=0} = 0,$$

$$(ii)\; H^{i}_{I_i I_j} = 0, \tag{4-12}$$

where $i,\ j = 1,\ 2.$

Condition (4-12 *(i)*) says that the Hamiltonians H^i, which is maximized with respect to u_i, is independent of the state variables I_j. Condition (4-12 *(ii)*) says that H^i is linear in the state variables I_i and I_j. These two properties are in fact satisfied by the Hamiltonians of this DDQG model. Hence, the Hamiltonians of the game are state separable. The implications of state separability are that each u_i is expressed only in terms of costate/adjoint variables and that the costate/adjoint equations do not contain the state variables. This means that the determination of the controls and the cosate/adjoint variables is separated from determination of the state variables. The optimality conditions of (4-4c) - (4-10) for this DDQG model show the state separability. Thus, *the open-loop solutions coincide with the feedback solutions*.

Nash and Stackelberg Strategies in the DDQG Model

The optimal solutions to the inventory problems are appealing since they provide rich insights for explanation and/or prediction about the system under study. In addition, they offer insights for operational empirical analysis. To do so, condition (4-5) will be examined first. Explicit solutions to (4-5) depend upon the explicit demand functional forms. From the point of view of player i, conditions (4-5) may be interpreted as: "*If I can estimate correctly demand and costs and how my rival will react to my quantity decision, my optimal decision must satisfy conditions (4-5) - (4-10).*" In practice, however, the task of estimating behavior parameters is undoubtedly difficult.

The presence of parameter CV may appear to create an ambiguity about the meaning of optimal policy, since any value of CV may fit any decision. Some authors suggest to consider this parameter in terms of solution concepts. In Chapter 2, for example, the Nash and Stackelberg strategies/solution concepts have been discussed. Lack of information to both players, which may lead to *simultaneity* and *independence* of the players in making their decision, is one of the reasons usually suggested to justify Nash strategies. The Nash strategies to the DDQG problem (sometimes called Nash-Cournot or Nash-in-quantity) is obtained when the CVs in quantity, γ_{ij}, are all equal to zero. This means that equation (4-5) can be written as

$$p_i = \frac{MC_i + \lambda_i^i}{(1 + \frac{1}{\eta_{ii}})} \tag{4-13}$$

The Stackelberg (in this case, Stackelberg-in-quantity) strategies when player i behaves as the leader means that player i will maximize its profit function along the player j's reaction function (which represents player j's best/optimal reply). This further implies that player j's CV, γ_{ji}, equals zero. The CV of Player i is derived from player j's optimization:

$$p_j = \frac{MC_j + \lambda_j^j}{(1 + \frac{1}{\eta_{jj}})} \tag{4-14}$$

Conditions (4-5) and (4-14) will stipulate the Stackelberg strategies for both players. The explicit solution, however, depends upon the functional form of the demand equation. Hence, if we know the inverse demand equation p_j, the CV of the leader/player i *(i.e.,* $\gamma_{ij} = \partial u_j/\partial u_i$) can be derived from (4-14). This CV is then substituted into (4-5) to obtain the Stackelberg strategy for player i.

One may reject the above idea of solution concepts and suggest that the behavioral aspect is a question of fact and hence needs to be resolved in empirical testing. The problem is a lack of supporting economic theory to develop a correct CV measurement. Needless to say, it is too early to conclude that, a particular CV model, Nash or Stackelberg is the correct answer.

Instead of considering CVs as behavioral parameters, some suggested an interpretation of CVs as the measures of market power (see for examples, Karp and Perloff, 1993; and Appelbaum, 1982), in the sense of the gap between marginal cost and price. The case of homogeneous products and static version of (4-5) (so we do not have to worry about the costate variables for a moment) may give a *better* form for this interpretation. Competitive market implies $\eta_{ii} = \eta_{jj} = \infty$ and $\gamma_{ij} = \gamma_{ji} = 0$ (price equals marginal cost). When both players act as monopolists, then $\gamma_{ij} = \gamma_{ji} = 0$ and each faces its individual downward sloping inverse demand function.

General Conjectural Variations (CV) for the DDQG Model

Instead of focusing on calculating optimal decisions, one may be interested in estimating the CVs. Provided that cost and demand functions can be correctly specified, conditions (4-5) to (4-10) provide a method for empirical analysis to investigate the *actual* strategic behavior of Florida and Brazilian producers in the OJ market. Equation (4-5) may be rearranged to obtain player *i*'s CV about the other player (player *j*):

$$\gamma_{ij} = \frac{MC_i + \lambda_i^i - p_i\left(\frac{\eta_{ii} + 1}{\eta_{ii}}\right)}{\left(\frac{p_i}{\eta_{ij}} \frac{u_i}{u_j} - \lambda_j^i\right)} \tag{4-15}$$

This CV conceptually measures how player *i* conjectures that player *j* will react dynamically to his/her quantity decision over the planning period. It may be still debatable whether the players make conjectures in terms of quantity, and hence may result in zero conjecture. Nevertheless, any *value* of conjecture, in terms of a particular control/strategic variable (say, quantity), under certain regularity condition, may be translated into another *value* of conjectures in terms of different control/strategic variables (for example price) (see *e.g.*, Krouse, 1990). Thus, Nash (-in-quantity) strategies do not necessarily mean a *naivete* of the player who fails to take into account his/her rival's decision.

Tacit/Implicit Price Collusion

Various possible outcomes may appear as optimal strategies from this DDQG. One appealing outcome of imperfect competition is *collusion* (in the sense that the prices are equal, not the merger of the firms). Even if players are engaged in a *noncooperative* competition, collusion (in this case usually called *tacit/implicit collusion*) is still possible. Homogeneity of the products, for example, may force the competing players to eventually sell them at the same common price. In a collusive price, optimality conditions (4-4c), (4-5) or (4-15) may be rearranged by substituting a common price p for p_i and p_j, and solve the equations for the two players. The results are not intutive, however, since it is still in very general form and various scenarios are possible. The following special cases are discussed, since they are appealing and commonly (at least theoretically) proposed as possible scenarios.

Nash strategies

Suppose that the players choose quantity as their strategies independently and simultaneously (and thus results in a Nash-in-quantity outcome), Nash strategies in a tacit collusion imply that (4-16) is satisfied. Substituting a common price p for p_i and p_j into (4-13) yields

$$\frac{\eta_{ii}}{(\eta_{ii} + 1)} (MC_i + \lambda_i^i) = \frac{\eta_{jj}}{(\eta_{jj} + 1)} (MC_j + \lambda_j^j) \tag{4-16}$$

Stackelberg strategies

The leader-follower scenario (in this case, Stackelberg-in-quantity) in a tacit collusion implies that (4-17) is satisfied. Assuming player i is the leader and player j is the follower, substituting the common price p into (4-14) for the follower and to (4-5) for the leader results in

$$\gamma_{ij} = \frac{(\frac{\eta_{jj}+1}{\eta_{jj}})(MC_i + \lambda_i^i) - (\frac{\eta_{ii}+1}{\eta_{ii}})(MC_j + \lambda_j^j)}{\frac{u_i}{u_j \eta_{ij}}(MC_j + \lambda_j^j) - \lambda_j^i (\frac{\eta_{jj}+1}{\eta_{jj}})}, \quad and$$

$$\gamma_{ji} = 0 \tag{4-17}$$

Static games for various scenarios may be derived as the limiting cases of this dynamic games by setting the adjoint/costate variables to zero.

Spatial Solutions to the DDQG Model

Certainly, the solutions to the spatial model depend on the assumptions about markets and cost structure. Here, assuming that the markets are independent and costs with respect to each market are also independent, generalization from the non-spatial to spatial solutions is, in principle, very straightforward. That is, the first necessary condition (4-4c) still holds and can be extended to:

$$p_{ik}(1 + \frac{1}{\eta_{ii}^{k}} + \frac{\gamma_{ij}^{k}}{\eta_{ij}^{k}} \frac{u_{ik}}{u_{jk}}) - MC_{ik} - \lambda_{i}^{i} - \lambda_{j}^{i}\, \gamma_{ij}^{k} = 0, \tag{4-18}$$

$$i \neq j, \quad i, j = F(lorida),\ B(razil) \quad for \quad k = U(S),\ E(urope).$$

Solving for γ_{ij}^{k} gives

$$\gamma_{ij}^{k} = \frac{MC_{ik} + \lambda_{i}^{i} - p_{ik}(\frac{\eta_{ii}^{k} + 1}{\eta_{ii}^{k}})}{(\frac{p_{ik}}{\eta_{ij}^{k}} \frac{u_{ik}}{u_{jk}} - \lambda_{j}^{i})}. \tag{4-19}$$

Equation (4-9) for the costate/adjoint variables still holds, and the relevant state equations/equation of motion are:

$$\dot{I}_{i}(t) = q_{i}(t) - \sum_{k} u_{ik}(t), \tag{4-20}$$

$$i = F(lorida),\ B(razil), \quad and \quad k = U(S),\ E(urope).$$

The foregoing results on the Nash, Stackelberg and price collusion in each market are applicable to the spatial model.

The Special Case of a Linear Inverse Demand Function

As can be seen from the foregoing discussion, the assumption of the specific functional form of demand is crucial to the explicit solutions to the models. The discussion will now investigate the explicit solutions in special case of linear inverse demand functions. For player *i*'s product in market *k*

$$p_{ik}(t) = \alpha^{k}_{0i}(t) + \alpha^{k}_{ii}(t)u_{ik}(t) + \alpha^{k}_{ij}(t)u_{jk}(t),$$
$$\text{for } k = U(S),\ E(urope). \tag{4-21}$$

where $\alpha_{0i}^{k}(t)$ represents factors, dependent on some exogenous variables[2] (which is assumed to be market specific), that may change over time, so that the own-price flexibilities of demand, $1/\eta_{ii}^{k}$, and the inverse cross-price flexibilities of demand, $1/\eta_{ij}^{k}$, are (time argument is supressed)

$$\frac{1}{\eta^{k}_{ii}} = \frac{\alpha^{k}_{ii}u_{ik}}{p_{ik}}, \qquad and \qquad \frac{1}{\eta^{k}_{ij}} = \frac{\alpha^{k}_{ij}u_{jk}}{p_{ik}} \tag{4-22}$$

The optimality condition of profit maximization gives the reaction function

$$u_{ik} = \frac{MC_{ik} + \lambda^{i}_{i} - \alpha^{k}_{0i} - \alpha^{k}_{ij}u_{jk} + \lambda^{i}_{j}\,\gamma^{k}_{ij}}{2\alpha^{k}_{ii} + \alpha^{k}_{ij}\gamma^{k}_{ij}} \tag{4-23}$$

It may be rearranged in the form of

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[2] For convenience, the exogenous variables are not expressed explicitly, since they have the same properties as α_{0i}^{k} in the derivation anyway.

$$\gamma_{ij}^{k} = \frac{MC_{ik} + \lambda_i^i - (\alpha_{0i}^k + 2\alpha_{ii}^k u_{ik} + \alpha_{ij}^k u_{jk})}{(\alpha_{ij}^k u_{ik} - \lambda_j^i)} \tag{4-24}$$

Nash and Stackelberg strategies can easily be calculated from either (4-23) or (4-24). The Nash strategies for player *i* are (Nash strategies for player *j* are identical):

$$(i)\ u_{ik} = \frac{MC_{ik} + \lambda_i^i - (\alpha_{0i}^k + \alpha_{ij}^k \mu_{jk})}{2\alpha_{ii}^k}; \quad \textit{or by eliminating } u_{jk} \textit{ gives}$$

$$(ii)\ u_{ik} = \frac{2\alpha_{jj}^k(MC_{ik} + \lambda_i^i - \alpha_{0i}^k) - \alpha_{ij}^k(MC_{jk} + \lambda_j^j - \alpha_{0j}^k)}{(4\alpha_{ii}^k\alpha_{jj}^k - \alpha_{ij}^k\alpha_{ji}^k)} \tag{4-25}$$

To derive the Stackelberg strategies (assuming player *i* is the leader), player *i*'s CV needs to be defined, which can be derived from the optimal solution for player *j* (equation (4-25 *(i)*) for *j*, since $\gamma_{ji} = 0$):

$$u_{jk} = \frac{MC_{jk} + \lambda_j^j - \alpha_{0j}^k - \alpha_{ji}^k u_{ik}}{2\alpha_{jj}^k} \tag{4-26}$$

and since the costs are linear in *u*, player *i*'s CV about player *j* obtained by deriving (4-26) with respect to u_{ik}, *i.e.*, $\partial u_{jk}/\partial u_{ik}$, is

$$\gamma_{ij}^{k} = \frac{-\alpha_{ji}^{k}}{2\alpha_{jj}^{k}}. \tag{4-27}$$

Substituting (4-27) into (4-23) altogether with (4-26) solving the Stackelberg strategies:

$$u_{ik} = \frac{2\alpha_{jj}^{k}(MC_{ik} + \lambda_i^i - \alpha_{0i}^{k}) - \alpha_{ij}^{k}(MC_{jk} + \lambda_j^j - \alpha_{0j}^{k}) - \alpha_{ji}^{k}\lambda_j^i}{2(2\alpha_{ii}^{k}\alpha_{jj}^{k} - \alpha_{ij}^{k}\alpha_{ji}^{k})}; \quad and$$

$$u_{jk} = \frac{(4\alpha_{ii}^{k}\alpha_{jj}^{k} - \alpha_{ij}^{k}\alpha_{ji}^{k})(MC_{jk}+\lambda_j^j-\alpha_{0j}^{k}) - 2\alpha_{ji}^{k}\alpha_{jj}^{k}(MC_{ik}+\lambda_i^i-\alpha_{0i}^{k})+\alpha_{ij}^{k}\alpha_{ji}^{k}\lambda_j^i}{4\alpha_{jj}^{k}(2\alpha_{ii}^{k}\alpha_{jj}^{k} - \alpha_{ij}^{k}\alpha_{ji}^{k})}. \tag{4-28}$$

Back to the special case of tacit collusion, linear demand curves provide some analytical results. First, general conditions for tacit collusion with linear demand curves can be obtained from an algebraic manipulation of (4-20) by substituting a common price p, which results, for player i (and identically for player j) in

$$u_{ik} = \frac{(\alpha_{jj}^{k} + \alpha_{ji}^{k}\gamma_{ji}^{k})u_{jk} + (MC_{ik} + \lambda_i^i) - (MC_{jk} + \lambda_j^j) + \lambda_j^i\gamma_{ij}^{k} - \lambda_i^j\gamma_{ji}^{k}}{(\alpha_{ii}^{k} + \alpha_{ij}^{k}\gamma_{ij}^{k})} \tag{4-29}$$

Thus, Nash strategies in a tacit collusion yield

$$u_{ik} = \frac{\alpha_{jj}^{k}u_{jk} + (MC_{ik} + \lambda_i^i) - (MC_{jk} + \lambda_j^j)}{\alpha_{ii}^{k}}. \tag{4-30}$$

The Stackelberg strategies in a tacit collusion can be derived from (4-27) and (4-29)

$$u_{ik} = \frac{2\alpha_{jj}^{k}(\alpha_{jj}^{k}u_{jk} + (MC_{ik} + \lambda_i^i) - (MC_{jk} + \lambda_j^j)) - \lambda_j^i\alpha_{ji}^{k}}{(2\alpha_{ii}^{k}\alpha_{jj}^{k} - \alpha_{ij}^{k}\alpha_{ji}^{k})}. \tag{4-31}$$

When the products are homogeneous and Nash strategies are employed in a collusive price in a particular market, it can shown from (4-30) that the relative efficiency between the duopolists determines their market shares: the more efficient player the higher his/her market share. In Stackelberg strategies, of course, the quantity leader likely gets the higher market share. Even if the two are equally efficient and if λ_j^i is negligable, the leader gets twice as much of the follower does.

$$u_{ik} = 2u_{jk} + \frac{2}{\alpha_1^k}((MC_{ik} + \lambda_i^i) - (MC_{jk} + \lambda_j^j)). \tag{4-32}$$

(α_1^k *is the slope of the inverse demand function*)

The Dynamic Duopoly Price Game (DDPG) Model

As mentioned before, price is one of the most significant strategic variables in competition. It has immediate effects on revenues, as a medium to communicate to buyers, and at the same time as a medium to react to rival's strategies. Pricing is perhaps the most realistic strategic variable for the short-run planning, since sellers eventually have to determine the price of their products.

Nonspatial Solutions to the DDPG Model

The general solutions for the price game similar to the above analysis is investigated. In this model, price is now the control instrument used by each player to maximize his/her discounted profit. Using the same current value Hamiltonians of (4-3) to derive the first order necessary conditions (with respect to price) yields

$$\frac{\partial H^i}{\partial p_i} = u_i + p_i \frac{\partial u_i}{\partial p_i} + p_i \left(\frac{\partial u_i}{\partial p_j} \frac{\partial p_j}{\partial p_i}\right) - \frac{\partial C_i}{\partial u_i} \left(\frac{\partial u_i}{\partial p_i} + \frac{\partial u_i}{\partial p_j} \frac{\partial p_j}{\partial p_i}\right)$$

$$- \lambda_i^i \left(\frac{\partial u_i}{\partial p_i} + \frac{\partial u_i}{\partial p_j} \frac{\partial p_j}{\partial p_i}\right) - \lambda_j^i \left(\frac{\partial u_j}{\partial p_i} + \frac{\partial u_j}{\partial p_j} \frac{\partial p_j}{\partial p_i}\right) = 0. \qquad (4\text{-}33)$$

Rearranging gives

$$u_i \left[1 + \left(\frac{\partial u_i}{\partial p_i} \frac{p_i}{u_i}\right) + \left[\left(\frac{\partial u_i}{\partial p_j} \frac{p_j}{u_i}\right) \left(\frac{\partial p_j}{\partial p_i}\right) \frac{p_i}{p_j}\right]\right]$$

$$- MC_i \left[\left(\frac{\partial u_i}{\partial p_i} \frac{p_i}{u_i}\right) \frac{u_i}{p_i} + \left(\frac{\partial u_i}{\partial p_j} \frac{p_j}{u_i}\right) \left(\frac{\partial p_j}{\partial p_i}\right) \frac{u_i}{p_j}\right]$$

$$- \lambda_i^i \left[\left(\frac{\partial u_i}{\partial p_i} \frac{p_i}{u_i}\right) \frac{u_i}{p_i} + \left(\frac{\partial u_i}{\partial p_j} \frac{p_j}{u_i}\right) \left(\frac{\partial p_j}{\partial p_i}\right) \frac{u_i}{p_j}\right]$$

$$- \lambda_j^i \left[\left(\frac{\partial u_j}{\partial p_i} \frac{p_i}{u_j}\right) \frac{u_j}{p_i} + \left(\frac{\partial u_j}{\partial p_j} \frac{p_j}{u_j}\right) \left(\frac{\partial p_j}{\partial p_i}\right) \frac{u_j}{p_j}\right] = 0. \qquad (4\text{-}34)$$

Define η_{ii}, η_{ij} or η_{ji}, and ρ_{ij} as, respectively, the own-price elasticity of demand, the cross-price elasticity of demand, and the CV in-price of player i about player j (*i.e.*, $\partial p_j/\partial p_i$). The own- and cross- costate variables λ_i^i and λ_j^i are as in DDQG model. It follows that

$$u_i \left(1 + \eta_{ii} + \eta_{ji}\rho_{ij}\frac{p_i}{p_j}\right) - \left(\eta_{ii}\frac{u_i}{p_i} + \eta_{ji}\,\rho_{ij}\frac{u_i}{p_j}\right)(MC_i + \lambda_i^i)$$
$$- \lambda_j^i \left(\eta_{ij}\frac{u_j}{p_i} + \eta_{jj}\rho_{ij}\frac{u_j}{p_j}\right) = 0,$$
$$where \qquad i \neq j. \tag{4-35}$$

Equation (4-38) can be rearranged as CV-in-price:

$$\rho_{ij} = \frac{\eta_{ii}\dfrac{u_i}{p_i}(MC_i + \lambda_i^i) - u_i(1 + \eta_{ii}) + \lambda_j^i\eta_{ij}\dfrac{u_j}{p_i}}{\dfrac{u_i}{p_j}\eta_{ji}(p_i - MC_i - \lambda_i^i) - \eta_{jj}\,\lambda_j^i\,\dfrac{u_j}{p_j}}, \tag{4-36}$$
$$where \quad i \neq j.$$

Equations (4-35) or (4-36) together with (4-9) and (4-10) are the necessary conditions for optimal solutions for the nonspatial open-loop DDPG model. Again, assuming that the H^i are jointly concave in p_i and I_i, these necessary conditions are also sufficient. It need not be shown again, that these open-loop solutions are also feedback solutions to this DDPG model.

Spatial Solutions to the DDPG Model

Again, since it is assumed that markets are independent, and the costs, with respect to each market, are also independent, the spatial model of this price game, can be derived as follows.

The first order conditions, an extention of (4-35) are

$$u_{ik}\left(1+\eta_{ii}^{k}+\eta_{ji}^{k}\rho_{ij}^{k}\frac{p_{ik}}{p_{jk}}\right)-\left(\eta_{ii}^{k}\frac{u_{ik}}{p_{ik}}+\eta_{ji}^{k}\rho_{ij}^{k}\frac{u_{ik}}{p_{jk}}\right)(MC_{ik}+\lambda_{i}^{i})$$

$$-\lambda_{j}^{i}\left(\eta_{ij}^{k}\frac{u_{jk}}{p_{ik}}+\eta_{jj}^{k}\rho_{ij}^{k}\frac{u_{jk}}{p_{jk}}\right)=0,$$

$$for \quad k=U(S),\ E(urope), \quad and \quad i\neq j \tag{4-37}$$

or

$$\rho_{ij}^{k}=\frac{\eta_{ii}^{k}\frac{u_{ik}}{p_{ik}}(MC_{ik}+\lambda_{i}^{i})-u_{ik}(1+\eta_{ii}^{k})+\lambda_{j}^{i}\eta_{ij}^{k}\frac{u_{jk}}{p_{ik}}}{\frac{u_{ik}}{p_{jk}}\eta_{ji}^{k}(p_{ik}-MC_{ik}-\lambda_{i}^{i})-\eta_{jj}^{k}\lambda_{j}^{i}\frac{u_{jk}}{p_{jk}}}, \tag{4-38}$$

$$for \quad k=U(S),\ E(urope), \quad and \quad i\neq j.$$

The optimality condition (4-37) or (4-38), the costate/adjoint condition (4-9) and the state equation/equation of motion (4-16) are the solutions to the spatial DDPG model.

The Special Case of a Linear Demand Function

Any particular strategies such as Nash (in this case sometimes called Nash-Bertrand or Nash-in-price) or Stackelberg-in-price strategies can be derived similarly to the analysis in quantity game, which in principle, depends upon particular demand and cost functional forms. Let the demand function for player *i*'s product in market *k* have the form of

$$\begin{aligned} u_{ik}(t) &= \beta_{0i}^{k}(t) + \beta_{ii}^{k}(t)p_{ik}(t) + \beta_{ij}^{k}(t)p_{jk}(t), \\ &\quad for \quad k = U(S),\ E(urope). \end{aligned} \tag{4-39}$$

so that own-price elasticities of demand, η_{ii}^{k}, and cross-price elasticities of demand, η_{ji}^{k}, are (time argument *t* is supressed)

$$\begin{aligned} \eta_{ii}^{k} &= (\beta_{ii}^{k} + \beta_{ij}^{k}\rho_{ij}^{k})\ \frac{p_{ik}}{u_{ik}}; \qquad and \\ \eta_{ji}^{k} &= (\beta_{ij}^{k} + \beta_{ii}^{k}\rho_{ji}^{k})\ \frac{p_{jk}}{u_{ik}} \end{aligned} \tag{4-40}$$

Equations ((4-37) and (4-40)) or ((4-33) and (4-39)) gives the reaction function for player *i* in term of player *j*'s price

$$p_{ik} = \frac{(\beta_{ii}^{k} + \beta_{ij}^{k}\ \rho_{ij}^{k})\ (MC_{ik} + \lambda_{i}^{i}) - \beta_{0i}^{k} - \beta_{ij}^{k}\ p_{jk} + \lambda_{j}^{i}(\beta_{ji}^{k} + \beta_{jj}^{k}\ \rho_{ij}^{k})}{2\beta_{ii}^{k} + \beta_{ij}^{k}\ \rho_{ij}^{k}} \tag{4-41}$$

$$for \quad k = U(S),\ E(urope), \qquad and \qquad i \neq j.$$

The Nash-Bertrand (Nash-in-price) strategy for player *i* (and identically for player *j*) is:

$$p_{ik} = \frac{2\beta_{ii}^{k}\beta_{jj}^{k}(MC_{ik} + \lambda_{i}^{i}) - \beta_{ij}^{k}\beta_{jj}^{k}(MC_{jk} + \lambda_{j}^{j}) - 2\,\beta_{0i}^{k}\beta_{jj}^{k} + \beta_{0j}^{k}\beta_{ij}^{k}}{4\beta_{ii}^{k}\beta_{jj}^{k} - \beta_{ij}^{k}\beta_{ji}^{k}}$$

$$+ \frac{2\beta_{ji}^{k}\beta_{jj}^{k}\lambda_{j}^{i} - (\beta_{ij}^{k})^{2}\lambda_{i}^{j}}{4\beta_{ii}^{k}\beta_{jj}^{k} - \beta_{ij}^{k}\beta_{ji}^{k}} \tag{4-42}$$

Suppose that player *i* is the leader and *j* is the follower, then $\rho_{ji}^{k} = 0$ and (4-41) gives $\rho_{ij}^{k} = -\beta_{ji}^{k}/2\beta_{jj}^{k}$. The non-collusive Stackelberg-in-price strategy for the follower is

$$p_{jk} = \frac{2\beta_{ii}^{k}\beta_{jj}^{k}(MC_{jk} + \lambda_{j}^{j}) - \beta_{ji}^{k}\beta_{ii}^{k}(MC_{ik} + \lambda_{i}^{i}) - 2\beta_{0j}^{k}\beta_{ii}^{k} + \beta_{0i}^{k}\beta_{ji}^{k}}{4\,\beta_{ii}^{k}\beta_{jj}^{k} - \beta_{ij}^{k}\beta_{ji}^{k}}$$

$$+ \frac{2\beta_{ij}^{k}\beta_{ii}^{k}\lambda_{i}^{j} - (\beta_{ji}^{k})^{2}\lambda_{j}^{i}}{4\beta_{ii}^{k}\beta_{jj}^{k} - \beta_{ij}^{k}\beta_{ji}^{k}} \tag{4-43}$$

and Stackelberg-in-price strategy for the leader is

$$p_{ik} = \frac{[D_k^2 - A_k(D_k + 4B_k)](MC_{ik} + \lambda_i^i) + 2B_k^2(c_{jk} + \lambda_j^j) + 4B_kE_k}{D_k^2}$$

$$\frac{- 2(B_kF_k + D_kE_k) + B_k\lambda_j^i(D_k + B_k) - 2B_k(\beta_{ij}^k)^2\lambda_i^j}{D_k^2} \tag{4-44}$$

where

$$A_k = \beta_{ii}^k\beta_{jj}^k; \qquad B_k = \beta_{ij}^k\beta_{jj}^k; \qquad E_k = \beta_{\alpha}^k\beta_{jj}^k$$

$$F_k = \beta_{0j}^k; \qquad \mathit{and} \qquad D_k = 4\,\beta_{ii}^k\beta_{jj}^k - \beta_{ij}^k\beta_{ji}^k.$$

Tacit price collusion in the DDPG model resulting from profit maximizing behavior is possible depending on the relative marginal costs and the players' conjectures. With Nash strategies, collusion is possible if (for player *i*, and similarly for player *j*)

$$p = \frac{\beta_{ii}^k(MC_{ik} + \lambda_i^i) - \beta_{0i}^k + \beta_{ji}^k\lambda_j^i}{2\,\beta_{ii}^k - \beta_{ij}^k}. \tag{4-45}$$

If both λ_j^i and λ_i^j are negligible, in the homogeneous case (4-45) implies that the marginal costs of the two players are the same. In Stackelberg strategies, the conditions are

For player i (leader) :

$$(i)\ p = \frac{(\beta^{k}_{ii}(2\beta^{k}_{jj}+\beta^{k}_{ji})-\beta^{k}_{ij}\beta^{k}_{ji})(MC_{ik}+\lambda^{i}_{i})-(2\beta^{k}_{jj}+\beta^{k}_{ji})\beta^{k}_{0i}+\lambda^{i}_{j}(\beta^{k}_{ji}(2\beta^{k}_{jj}\beta^{k}_{ji}-\beta^{k}_{i}}{(2\beta^{k}_{ii}+\beta^{k}_{ij})(2\beta^{k}_{jj}-\beta^{k}_{ji})-\beta^{k}_{ij}\beta^{k}_{ji})} \tag{4-46}$$

and the follower equates its price such that

$$(ii)\ p = \frac{(\beta^{k}_{jj}+\beta^{k}_{ji})(MC_{jk}+\lambda^{j}_{j})-\beta^{k}_{0j}+\lambda^{j}_{i}(\beta^{k}_{ij}+\beta^{k}_{ii})}{2\beta^{k}_{jj}}.$$

Equivalent Strategies in DDQG and DDPG Models

As discussed earlier in Chapter 2, many rejected Nash-Cournot strategies as proper solutions to duopoly (or oligopoly in general) competitions. One reason is that Nash-Cournot or Nash-Bertrand strategies may be interpreted as solutions when each player chooses his/her strategy which maximizes his/her payoff when the strategy selected by the rival is held constant. This assumed lack of interaction may appear contrary to the widely believed interdependence of duopoly. This is not necessarily true. Provided that relationship between strategies is well-defined and suitably regular (for examples, linear relationships between the quantity sold, u_{ik}, in the DDQG model and the price, p_{ik}, in the DDPG model), zero conjectures (Nash) in one strategy may correspond to nonzero conjectures in the other strategy variable (for more details, see Krouse, 1990). The proof of this correspondence is provided in Appendix C.

CHAPTER 5
EMPIRICAL RESULTS AND DISCUSSION

Specification of the Empirical Models

Different approaches to developing empirical models utilizing the conjectural variation concept have been employed in studies of imperfect market competitions. At least three general approaches can be found. First, an *independent* estimation approach. In this approach, estimation of the demand function is independently carried out from the estimation of parameters of CVs, cost, and reaction functions. Particular behavioral parameters of CVs are usually hypothesized. Iwata (1974) and Rogers (1989) are among those who employed this approach. One of the problems with this approach is that the estimation problem deals with the simultaneous equations. In a perfectly competitive market, reaction functions are not relevant, and hence the demand equation can be estimated in isolation. In contrast, it is usually believed that there exist some form of interdependences among the players in imperfect market competitions. At least two types of equations are involved in an imperfect competition model: (inverse) demand and reaction functions. In the quantity game (DDQG) model, for example, quantity variables u_i, u_j, in the inverse demand functions are now endogenous. Therefore, these functions should be estimated simultaneously with the reaction functions. Otherwise, ordinary least squares (OLS) estimates are biased and inconsistent.

The second approach is a *purely* optimization approach. Instead of hypothesizing behavioral parameters CVs, one may have *a priori* knowledge about or (usually) simply assume particular values, say zero (if all players employ Nash strategies), for these parameters. The optimization algorithm may be implemented based upon the assumed solution concepts/behavioral parameters (and the numerical results may be compared with the observed values). One may also want to implement this approach in conjunction with a particular criterion, say minimizing deviation from the observed values.[1] Among the empirical studies utilizing this approach are Karp and McCalla (1983), Kolstad and Burris (1986), Murphy *et al.* (1989), and Karp and Perloff (1993). Karp and McCalla assumed the Nash-Cournot conjectures in the solution to their model of the world corn market and compared the numerical results with the observed values. Kolstad and Burris also assumed the Nash-Cournot solutions in their model of the international wheat market and also compared them with the observed values. Murphy *et al.* employed the Nash-Cournot as the solution concepts to derive analytical and numerical results (but not for the purpose of comparing them with the observed values). Karp and Perloff calculated numerical results based on different assumptions on the behavioral parameters (such as collusive and price taking behavior). Breton *et al.* (1988) suggested a sequential Stackelberg algorithm for a two-person game, but did not apply it to an actual problem. Cyert and DeGroot (1970) and Coughlan and Mantrala (1992, 1994) also developed dynamic pricing models in which players decide alternately. Backward recursion techniques of dynamic programming are usually employed in this approach.

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[1] This is not usually performed, however, since the procedure would generally be difficult to develop.

This approach seems appealling. However, there are some problems with this approach. Provided that the assumed behavioral parameters of CVs are accurate, then the approach can be utilized to derive the projected optimal decisions/strategies in the competition.[2] But if the major issue is to evaluate the degree of competition itself and/or to evaluate the performance of the model, then this approach may not be appropriate. In general, the accuracy of the results is sensitive to the assumptions on the conjectural variation parameters. One of the major issues in imperfect market competitions is investigating these behavioral parameters. Having the knowledge of the conduct (or market power/degree of competition) may be able to address some important issues in an imperfect competition. Thus, assuming particular behavioral parameters of CVs in investigating imperfect markets is certainly a crucial step. Various scenarios of behavioral parameters/solution concepts may be employed. But this is certainly not a simple task to do. Numerical results would need to be derived for each scenario. Moreover, only qualitative evaluation can be made about the numerical results derived from those scenarios with this approach (unless other certain criteria, such as minimizing deviation from the observed values, are also imposed). Similar to the problem noted with the first approach, the demand functions used in this approach also should be estimated simultaneously with the reaction functions. Combining an estimation of a system of simultaneous equations with dynamic optimization algorithms is certainly not a simple task to do.

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[2] Thus, this may be viewed as a *normative* approach suggesting how optimal strategies should be employed in a particular situation/conduct represented by the CVs.

The third approach is that of estimating a system of simultaneous equations (of (inverse) demand and reaction functions). Like the first method, this is a *positive* approach in which the main purpose is to answer *'what is'*, rather than *'what should be'* questions about the system under study. Applebaum (1982) and Liang (1989) are among those who have utilized this approach. Since independent variables in demand functions are considered endogenous, demand functions are estimated simultaneously with reaction functions. The advantage of this approach is that if we are able to specify the required functions and have sufficient data, we may be able to estimate the parameters of the model properly and obtain the statistical properties of the results. We may hypothesize some specific values for the CVs that represent particular behavioral parameters or degree of competition/market power (such as competitive or monopoly) and apply statistical tests. Statistical computer software is available for this purpose. However, there are some disadvantages with this approach. Specification and estimation of a simultaneous equation model is not an easy task to do and is generally time and data demanding. Nevertheless, this approach seems to best suit the empirical purpose of this study, and thus is utilized for the empirical analysis.

As discussed previously, assumption about *valuation* of the rival's inventory system (4-6) is important to the outcome. Throughout the empirical analysis, a_{ij} is assumed to be zero for $i \neq j$. Thus, the relevance/importance of year-ending inventory may be defined as

$$\Lambda_{ij} = \begin{cases} 1, & i = j, \\ 0, & otherwise \end{cases} \tag{5-1}$$

The rationale in equation (5-1) is that it is assumed that there is no cooperation between the players. In addition, no player has control on its rival's year-ending inventory and so at the end of the game/final planning period (*T*), each player's year-ending inventory condition has no relevance to its rival.

The DDQG Model

With the DDQG model, it is assumed that each player maximizes his/her profit by varying quantity. The optimality conditions derived previously provide the best/optimal strategies for each player over the planning horizon of one year, given that demand and cost functions, and market power/behavioral parameters CVs (*i.e.*, γ_{ij} and γ_{ji}) can be correctly specified. The empirical difficulty is that the CVs are unknown. One may hypothesize values for the CVs that correspond with particular behavior/market power. If the empirical estimates of the CVs give values consistent with given behavior or a particular level of market power, then, strong empirical evidence exists in favor of the hypothesis that this behavior is followed (or that each player has exercised his/her corresponding market power).

For the DDQG Model, the complete equations (time argument is suppressed) to be estimated are (see *Key to Symbols* for notation)

(1) Linear inverse demand functions (see (4-23)):

$$
\begin{aligned}
&\textit{For Florida in the U.S.}\\
&p_{F1} = \alpha_{0F}^{1} + \alpha_{FF}^{1} u_{F1} + \alpha_{FB}^{1} u_{B1} + \alpha_{F2}^{1} IC_{1} + \alpha_{F3}^{1} AJ_{1} + \epsilon_{1},\\
&\textit{For Brazil in the U.S.}\\
&p_{B1} = \alpha_{0B}^{1} + \alpha_{BB}^{1} u_{B1} + \alpha_{BF}^{1} u_{F1} + \alpha_{B2}^{1} IC_{1} + \alpha_{B3}^{1} ERB + \epsilon_{2},\\
&\textit{For Florida in Europe}\\
&p_{F2}(1 + \tau_{2}) = \alpha_{0F}^{2} + \alpha_{FF}^{2} u_{F2} + \alpha_{FB}^{2} u_{B2} + \alpha_{F2}^{2} IC_{2} + \alpha_{F3}^{2} ER + \epsilon_{3},\\
&\textit{For Brazil in Europe}\\
&p_{B2}(1 + \tau_{2}) = \alpha_{0B}^{2} + \alpha_{BB}^{2} u_{B2} + \alpha_{BF}^{2} u_{F2} + \alpha_{B2}^{2} ICG_{2} + \alpha_{B3}^{2} ER + \epsilon_{4}.
\end{aligned}
\tag{5-2}
$$

(2) Quantity reaction functions (see (4-25)):

$$
\begin{aligned}
&\textit{For Florida in the U.S.}\\
&u_{F1} = \frac{1}{2\,\alpha_{FF}^{1} + \alpha_{FB}^{1}\gamma_{FB}^{1}}[(MC_{F1} + \lambda_{F}^{F}) - (\alpha_{FB}^{1} u_{B1} + \alpha_{0F}^{1} + \alpha_{F2}^{1} IC_{1}\\
&\qquad\qquad + \alpha_{F3}^{1} AJ_{1} + \alpha_{F4}^{1} SF_{1})] + \epsilon_{5},\\
&\textit{For Brazil in the U.S.}\\
&u_{B1} = \frac{1}{2\,\alpha_{BB}^{1} + \alpha_{BF}^{1}\gamma_{BF}^{1}}[(MC_{B1} + \tau_{1} + \lambda_{B}^{B}) - (\alpha_{BF}^{1} u_{F1} + \alpha_{0B}^{1}\\
&\qquad\qquad + \alpha_{B2}^{1} IC_{1} + \alpha_{B3}^{1} ERB + \alpha_{F4}^{1} SB)] + \epsilon_{6},\\
&\textit{For Florida in Europe}\\
&u_{F2} = \frac{1}{2\,\alpha_{FF}^{2} + \alpha_{FB}^{2}\gamma_{FB}^{2}}[(MC_{F2} + \lambda_{F}^{F})(1 + \tau_{2}) - (\alpha_{FB}^{2} u_{B2} + \alpha_{0F}^{2}\\
&\qquad\qquad + \alpha_{F2}^{2} IC_{2} + \alpha_{F3}^{2} ER + \alpha_{F4}^{2} TT + \alpha_{F5}^{2} SF_{2})] + \epsilon_{7},\\
&\textit{For Brazil in Europe}\\
&u_{B2} = \frac{1}{2\,\alpha_{BB}^{2} + \alpha_{BF}^{2}\gamma_{BF}^{2}}[(MC_{B2} + \lambda_{B}^{B})(1 + \tau_{2}) - (\alpha_{BF}^{2} u_{F2} + \alpha_{0B}^{2}\\
&\qquad\qquad + \alpha_{B2}^{2} ICG_{2} + \alpha_{B3}^{2} ER + \alpha_{B4}^{2} TT + \alpha_{B5}^{2} SB_{2})] + \epsilon_{8},
\end{aligned}
\tag{5-3}
$$

(3) Rate of change in inventory:

$$\dot{I}_F = q_F - (u_{F1} + u_{F2} + u_{FROW}),$$
$$\dot{I}_B = q_B - (u_{B1} + u_{B2} + u_{BROW}). \qquad (5\text{-}4)$$

The DDPG Model

An empirical analysis with the DDPG model is carried out to estimate price conjectural variations as measures of the degree of price competition in the world FCOJ market. An econometric model is developed which is comprised of a simultaneous equation system of product demand and price reaction functions. A linear-differentiated product demand (4-42) is assumed in each market. Thus, price reaction functions are estimated where the price response of a player to the other player's price is evaluated in conjunction with the anticipated price response. In addition, own- and cross-price elasticities of demand are estimated along with the price conjectural variations.

The equations to be estimated in each market are individual demand functions (5-5), price reaction functions (5-6), and rate of change of inventory (see (5-4)). As in the DDQG model, the tariff rates (τ_k) are included in the DDPG model.

(1) Demand functions (similar to (4-42)):

For Florida in the U.S.

$$u_{F1} = \beta^{1}_{0F} + \beta^{1}_{FF}\, p_{F1} + \beta^{1}_{FB}\, p_{B1} + \beta^{1}_{F2} IC_{1} + \beta^{1}_{F3} PAJ_{1} + \nu_{1},$$

For Brazil in the U.S.

$$u_{B1} = \beta^{1}_{0B} + \beta^{1}_{BB}\, p_{B1} + \beta^{1}_{BF}\, p_{F1} + \beta^{1}_{B2} IC_{1} + \beta^{1}_{B3} ERB + \nu_{2},$$

(5-5)

For Florida in Europe

$$u_{F2} = \beta^{2}_{0F} + (1 + \tau_{2})(\beta^{2}_{FF}\, p_{F2} + \beta^{2}_{FB}\, p_{B2}) + \beta^{2}_{F2} IC_{2} + \beta^{2}_{F3} ER + \beta^{2}_{F42} TT + \nu_{3},$$

For Brazil in Europe

$$u_{B2} = \beta^{2}_{0B} + (1 + \tau_{2})(\beta^{2}_{BB}\, p_{B2} + \beta^{2}_{BF}\, p_{F2}) + \beta^{2}_{B2} ICG_{2} + \beta^{2}_{B3} ER + \beta^{2}_{B42} TT + \nu_{4}.$$

(2) Price reaction functions (see (4-44)):

For Florida in the U.S.

$$p_{F1} = \frac{\beta^{1}_{FF} + \beta^{1}_{FB}\rho^{1}_{FB}}{2\,\beta^{1}_{FF} + \beta^{1}_{FB}\rho^{1}_{FB}}[(MC_{F1} + \lambda^{F}_{F}) - (\beta^{1}_{FB}\, p_{B1} + \beta^{1}_{0F} + \beta^{1}_{F2} IC_{1}$$

$$+ \beta^{1}_{F3} PAJ_{1} + \beta^{1}_{F4} INF)] + \nu_{5},$$

For Brazil in the U.S.

$$p_{B1} = \frac{\beta^{1}_{BB} + \beta^{1}_{BF}\rho^{1}_{BF}}{2\,\beta^{1}_{BB} + \beta^{1}_{BF}\rho^{1}_{BF}}[(MC_{B1} + \lambda^{B}_{B} + \tau_{1}) - (\beta^{1}_{BF}\, p_{F1} + \beta^{1}_{0B}$$

$$+ \beta^{1}_{B2} IC_{1} + \beta^{1}_{B3} ERB + \beta^{1}_{F4} INB)] + \nu_{6},$$

(5-6)

For Florida in Europe

$$(1 + \tau_{2}) p_{F2} = \frac{\beta^{2}_{FF} + \beta^{2}_{FB}\rho^{2}_{FB}}{2\,\beta^{2}_{FF} + \beta^{2}_{FB}\rho^{2}_{FB}}[(1 + \tau)(MC_{F2} + \lambda^{F}_{F} - \beta^{2}_{FB}\, p_{B2}) - \beta^{2}_{0F}$$

$$- \beta^{2}_{F2} IC_{2} - \beta^{2}_{F3} ER - \beta^{2}_{F4} INF] + \nu_{7},$$

For Brazil in Europe

$$(1 + \tau_{2}) p_{B2} = \frac{\beta^{2}_{BB} + \beta^{2}_{BF}\rho^{2}_{BF}}{2\,\beta^{2}_{BB} + \beta^{2}_{BF}\rho^{2}_{BF}}[(1 + \tau_{2})(MC_{B2} + \lambda^{B}_{B} - \beta^{2}_{BF}\, p_{F2}) - \beta^{2}_{0B}$$

$$- \beta^{2}_{B2} ICG_{2} - \beta^{2}_{B3} ER - \beta^{2}_{B4} INB] + \nu_{8},$$

Data

The primary sources of data are those published by the Florida Department of Citrus (FDOC) and the Florida Citrus Processors Association (FCPA). The econometric analysis were conducted based on time series data for three seasons (1990/91 - 1992/93). FCOJ data for Florida (pack, movement, and inventory, aggregated from weekly data) are obtained from the seasonal statistical summary published by the FCPA. Since this study focuses only on bulk FCOJ, the most relevant price entering storage decisions is then the FOB/CIF or wholesale price. Moreover, from an economic point of view, FOB and CIF prices may serve as reference prices since they represent what the commodity can earn as an export or what it costs the economy as an import. Price (in US $ per thousands SSE gallons) and quantity (in thousands SSE gallons) data for Florida (domestic and export) and Brazilian FCOJ (except European prices) are obtained from the FDOC. Price in the U.S. is based on CIF Tampa. Price in Europe for Florida FCOJ is calculated from export values (FDOC), and for Brazilian FCOJ is from Foodnews (various issues). All European prices are adjusted by the 19% ad valorem tariff imposed by the EC on imported FCOJ. The only available data of monthly Brazilian FCOJ exports (by destination) and exchange rate are those started from the 1990/91 season. Therefore, estimation was based on only sample period of three seasons (190/91 - 1992/93). Monthly Brazilian FCOJ production data for these three seasons, however, were not available. Hence, monthly Brazilian FCOJ production is estimated based on previous (1983/84 - 1989/90) available data (CACEX/BANCO DO BRAZIL) (see Appendix D for detail).

There are four cost components important to the citrus industry (Ward and Kilmer, 1989): (1) production cost, (2) picking and hauling, (3) packing and/or processing, and (4) selling. Their report shows how the four cost elements have varied over time (1967-1984). Required cost data for this study, however, are not available. As mentioned before, the marginal cost for each player is assumed to be constant in this study. This assumption is not unrealistic. First, the study deals with a short-run horizon. It is generally true that in a short run, fewer factors can be varied. Hence, it is plausable to assume a constant marginal cost for a short run model. Thompson (1989, p. 232) also asserted that various empirical studies support that in the short run, a linear total cost function with constant marginal cost is the pattern that best seems to describe actual cost behavior over the *normal* operating range of output. Cost data are based on a comparative cost study by Muraro (1993). Florida cost data are adjusted according to the rate of change of specified costs in Southwest Florida (see Muraro *et al.*, 1994) during that period (*i.e.*, 2 percent increase in 1991/92, and 1 percent decrease in 1992/93 from the previous year cost). The shadow values of inventory, both for Florida and Brazil, are estimated based on a 3 percent discount rate. The shadow value of inventory for each player at the terminal planning period is approximated by its marginal cost of inventory. Monthly income data for the U.S. are based on disposable personal income (U.S. Department of Commerce) adjusted by estimated population (United Nation). For the European market, income data (GDP) in West Germany are used, and a monthly series is estimated based on quarterly data (OECD) adjusted by population (UN). Apple juice price and quantity in the U.S. market are based on Nielsen.

Numerical Results

Estimation of the empirical models were hampered by lack of appropriate data. Estimates were based on deflated price and income. Three-stage least square/3SLS, and full information likelihood/FIML were both employed to estimate the models. FIML never gave satisfactory results (always failed to converge). Three-stage least square was first used (without imposing the restrictions on parameter estimates) to obtain the starting values for the estimation of the simultaneous equation system. Then, the restrictions were imposed, the equations were reestimated by 3SLS to obtain all the parameters of the model (including the conjectures).

The DDQG Model

For the DDQG model, the empirical results show that the demand function estimates are fairly consistent (Table 5-1). Notation for the coefficient estimates correspond to those in equations (5-2) and (5-3). For both Florida demand equations in the U.S. and Europe, the intercept and own quantity estimates are statistically significant. Coefficients of income, however, have the wrong sign and are not significant. Coefficients of Brazilian FCOJ quantity are not significant. Correlations between Florida and Brazilian FCOJ, both in the U.S. and European markets are low ($r = 0.15$ in the U.S., and $r = 0.46$ in Europe). The other two exogenous variables, apple juice consumption (in the U.S.) and ECU (in Europe), are not significant. For the Brazil demand equations, only in Europe, the coefficients of quantity (both for Brazil and

Table 5-1 Estimates of the inverse demand functions for the DDQG model.

	Parameter estimate	Standard error	t-statistic
For Florida in the U.S.			
α_{0F}^{1} (intercept)	2282.960	721.492	3.164
α_{FF}^{1} (own quantity)	-.007	.001	-5.004
α_{FB}^{1} (Brazil quantity)	.001	.001	1.140
α_{F2}^{1} (income)	-.072	.062	-1.153
α_{F3}^{1} (apple juice)	.004	.005	.839
For Florida in Europe			
α_{0F}^{2} (intercept)	2617.360	1276.260	2.051
α_{F0}^{2} (own quantity)	-.107	.020	-5.292
α_{FB}^{2} (Brazil quantity)	-.002	.001	-1.878
α_{F2}^{2} (income/ECU)	-211.958	235.966	-.898
α_{F3}^{2} (ECU)	-.245	.508	-.482
For Brazil in the U.S.			
α_{0B}^{1} (intercept)	1664.190	917.075	1.815
α_{BB}^{1} (own quantity)	-.001	.001	-.940
α_{BF}^{1} (Florida quantity)	-.005	.002	-2.045
α_{B2}^{1} (income)	-.019	.080	-.239
α_{B3}^{1} (Brazilian ER)	-.002	.001	-1.571
For Brazil in Europe			
α_{0B}^{2} (intercept)	1842.880	1356.280	1.359
α_{BB}^{2} (own quantity)	-.006	.002	-2.874
α_{BF}^{2} (Florida quantity)	-.076	.034	-2.238
α_{B2}^{2} (income)	-.164	.384	-.427
α_{B3}^{2} (ECU)	.173	.335	.516

Florida) are significant. Income coefficients, both in the U.S. and Europe, also have negative sign. Other exogenous variables, Brazilian exchange rate (in the U.S.) and ECU (in Europe) are both insignificant.

Estimates of the quantity reaction functions are shown in Table 5-2. The joint parameter (Wald) test of each reaction function indicates that only the Florida reaction function in the European market is not statistically significant. For the Florida reaction function in the U.S. and the European market, the intercepts and costs appear to be important determinants. The Brazil reaction functions in the U.S. and the European market have significant cost coefficients, but its sign in the U.S. market is wrong. In the U.S. market, coefficients of the rival's quantity have the wrong sign, both for the Florida and the Brazil reaction functions.

Estimates of the quantity conjectures and seasonal and time (year) dummies are shown in Table 5-3. As can be seen time is an important determinant both in the Florida and Brazil reaction functions in the European market. In this market, the conjectural variation estimate indicates that Brazil employs the Nash conjecture. This is not completely surprising, since Brazil is the dominant supplier and thus behaves as if it is a monopolist. Tests of hypothesis for particular conjectural variations were conducted. The hypotheses of Nash-in-quantity (Nash-Cournot) conjectures for both players are rejected, both for the U.S. and European markets. For consistent constant conjecture hypothesis, only the Brazilian conjectural variation in the U.S. market (γ_{BF}^{1}) cannot be rejected. The hypotheses of quantity leadership (Stackelberg-in-quantity) for

Table 5-2 Estimates of the quantity reaction functions for the DDQG model.

	Parameter estimate	Standard error	t-statistic
For Florida in the U.S.			
$-\alpha_{0F}^{1}/(2\alpha_{FF}^{1}+\alpha_{FB}^{1}\gamma_{FB}^{1})$ (intercept)	550768.000	300134.000	1.835
$-\alpha_{FB}^{1}/(2\alpha_{FF}^{1}+\alpha_{FB}^{1}\gamma_{FB}^{1})$ (Brazil qty.)	.198	.192	1.032
$1/(2\alpha_{FF}^{1}+\alpha_{FB}^{1}\gamma_{FB}^{1})$ (marg. cost)	-241.252	81.326	-2.966
$-\alpha_{F2}^{1}/(2\alpha_{FF}^{1}+\alpha_{FB}^{1}\gamma_{FB}^{1})$ (income)	-17.263	18.532	-.932
$-\alpha_{F3}^{1}/(2\alpha_{FF}^{1}+\alpha_{FB}^{1}\gamma_{FB}^{1})$ (apple juice)	1.016	1.197	.849
For Florida in Europe			
$-\alpha_{0F}^{2}/(2\alpha_{FF}^{2}+\alpha_{FB}^{2}\gamma_{FB}^{2})$ (intercept)	41113.700	20469.800	2.009
$-\alpha_{FB}^{2}/(2\alpha_{FF}^{2}+\alpha_{FB}^{2}\gamma_{FB}^{2})$ (Brazil qty.)	-.037	.027	-1.361
$1/(2\alpha_{FF}^{2}+\alpha_{FB}^{2}\gamma_{FB}^{2})$ (marg. cost)	-15.708	6.504	-2.415
$-\alpha_{F2}^{2}/(2\alpha_{FF}^{2}+\alpha_{FB}^{2}\gamma_{FB}^{2})$ (income/ECU)	-3329.460	3392.940	-.981
$-\alpha_{F3}^{2}/(2\alpha_{FF}^{2}+\alpha_{FB}^{2}\gamma_{FB}^{2})$ (ECU)	-3.853	7.536	-.511
For Brazil in the U.S.			
$-\alpha_{0B}^{1}/(2\alpha_{BB}^{1}+\alpha_{BF}^{1}\gamma_{BF}^{1})$ (intercept)	-424773.000	282285.000	1.505
$-\alpha_{BF}^{1}/(2\alpha_{BB}^{1}+\alpha_{BF}^{1}\gamma_{BF}^{1})$ (Florida qty.)	1.167	.644	1.813
$1/(2\alpha_{BB}^{1}+\alpha_{BF}^{1}\gamma_{BF}^{1})$ (marg. cost)	255.243	117.619	2.170
$-\alpha_{B2}^{1}/(2\alpha_{BB}^{1}+\alpha_{BF}^{1}\gamma_{BF}^{1})$ (income)	4.900	20.286	.242
$-\alpha_{B3}^{1}/(2\alpha_{BB}^{1}+\alpha_{BF}^{1}\gamma_{BF}^{1})$ (Brazilian ER)	.495	.466	1.061
For Brazil in Europe			
$-\alpha_{0B}^{2}/(2\alpha_{BB}^{2}+\alpha_{BF}^{2}\gamma_{BF}^{2})$ (intercept)	248165.000	203949.000	1.217
$-\alpha_{BF}^{2}/(2\alpha_{BB}^{2}+\alpha_{BF}^{2}\gamma_{BF}^{2})$ (Florida qty.)	-10.281	6.617	-1.554
$1/(2\alpha_{BB}^{2}+\alpha_{BF}^{2}\gamma_{BF}^{2})$ (marg. cost)	-134.662	43.921	-3.066
$-\alpha_{B2}^{2}/(2\alpha_{BB}^{2}+\alpha_{BF}^{2}\gamma_{BF}^{2})$ (income)	-22.053	53.211	-.414
$-\alpha_{B3}^{2}/(2\alpha_{BB}^{2}+\alpha_{BF}^{2}\gamma_{BF}^{2})$ (ECU)	23.293	46.413	.502

Table 5-3 Estimates of the quantity conjectural variations, time and seasonal factors in the DDQG model.

	Parameter estimate	Standard error	t-statistic
For Florida in the U.S.			
γ_{FB}^1 (conjecture)	12.539	11.352	1.105
α_{FS}^1 (seasonality)	4.749	17.040	.279
For Brazil in the U.S.			
γ_{BF}^1 (conjecture)	-1.302	.673	-1.936
α_{BS}^1 (seasonality)	-84.359	58.774	-1.435
For Florida in Europe			
γ_{FB}^2 (conjecture)	-64.461	42.511	-1.516
α_{F4}^2 (time)	69.129	22.300	3.100
α_{FS}^2 (seasonality)	12.125	30.532	.397
For Brazil in Europe			
γ_{BF}^2 (conjecture)	-.059	.048	-1.219
α_{B4}^2 (time)	78.671	29.022	2.711
α_{BS}^2 (seasonality)	-9.569	33.459	-.286

for each player in both markets are also rejected. Inconsistencies in the reaction functions may indicate that there is no strong statistical evidence to support a constant quantity conjecture assumption employed by either Brazil or Florida in the two markets during the observed period.

The DDPG Model

The empirical results for the demand equations, reaction functions and conjectural parameters with the DDPG model are shown in Tables 5-4, 5-5, and 5-6, respectively.

Table 5-4 Estimates of the demand functions for the DDPG model.

	Parameter estimate	Standard error	t-statistic
For Florida in the U.S.			
β_{OF}^{1} (intercept)	169400.000	92083.500	1.840
β_{FF}^{1} (own price)	-72.369	22.916	-3.577
β_{FB}^{1} (Brazil price)	11.805	22.862	.516
β_{F2}^{1} (income)	-4.216	7.930	-.532
β_{F3}^{1} (apple j. price)	.510	7.977	.064
β_{F4}^{1} (seasonality)	3287.970	3766.880	.873
For Florida in Europe			
β_{OF}^{2} (intercept)	1629.530	2437.380	.669
β_{FF}^{2} (own price)	-.325	.570	-.571
β_{FB}^{2} (Brazil price)	.019	.138	.141
β_{F2}^{2} (income/ECU)	-241.801	417.366	-.579
β_{F3}^{2} (ECU)	-.211	.906	-.233
β_{F4}^{2} (seasonality)	919.883	21.776	4.344
β_{F5}^{2} (time)	814.827	96.665	8.429
For Brazil in the U.S.			
β_{OB}^{1} (intercept)	8900.490	52801.800	.169
β_{BB}^{1} (own price)	-40.725	16.667	-2.443
β_{BF}^{1} (Florida price)	1.359	11.744	.116
β_{B2}^{1} (income)	3.909	4.118	.949
β_{B3}^{1} (Brazilian ER)	-.098	.066	-1.494
β_{B4}^{1} (seasonality)	31236.800	3647.900	8.563
For Brazil in Europe			
β_{OB}^{2} (intercept)	65503.700	39556.300	1.656
β_{BB}^{2} (own price)	-2.723	11.277	-.241
β_{BF}^{2} (Florida price)	1.820	6.340	.287
β_{B2}^{2} (income)	-15.330	10.525	-1.456
β_{B3}^{2} (ECU)	-4.513	8.596	-.525
β_{B4}^{2} (seasonality)	13232.900	5366.500	2.466
β_{B5}^{2} (time)	20520.400	2342.730	8.759

Table 5-5 Estimates of the price reaction functions for the DDPG model.

	Parameter estimate	Standard error	t-statistic
For Florida in the U.S.			
$-\beta_{0F}^{1}/(2\beta_{FF}^{1}+\beta_{FB}^{1}\rho_{FB}^{1})$ (intercept)	-12362.100	22962.000	-.538
$-\beta_{FB}^{1}/(2\beta_{FF}^{1}+\beta_{FB}^{1}\rho_{FB}^{1})$ (Brazil price)	-.861	2.992	-.288
$(\beta_{FF}^{1}+\beta_{FB}^{1}\rho_{FB}^{1})/(2\beta_{FF}^{1}+\beta_{FB}^{1}\rho_{FB}^{1})$ (marg. cost)	6.281	10.126	.620
$-\beta_{F2}^{1}/(2\beta_{FF}^{1}+\beta_{FB}^{1}\rho_{FB}^{1})$ (income)	.308	.819	.375
$-\beta_{F3}^{1}/(2\beta_{FF}^{1}+\beta_{FB}^{1}\rho_{FB}^{1})$ (app. juice price)	-.037	.550	-.068
For Florida in Europe			
$-\beta_{0F}^{2}/(2\beta_{FF}^{2}+\beta_{FB}^{2}\rho_{FB}^{2})$ (intercept)	45974.300	77165.000	.166
$-\beta_{FB}^{2}/(2\beta_{FF}^{2}+\beta_{FB}^{2}\rho_{FB}^{2})$ (Brazil price)	.550	3.100	.177
$(\beta_{FF}^{2}+\beta_{FB}^{2}\rho_{FB}^{2})/(2\beta_{FF}^{2}+\beta_{FB}^{2}\rho_{FB}^{2})$ (marg. cost)	-8.179	52.285	-.156
$-\beta_{F2}^{2}/(2\beta_{FF}^{2}+\beta_{FB}^{2}\rho_{FB}^{2})$ (income/ECU)	-6821.970	41421.500	-.165
$-\beta_{F3}^{2}/(2\beta_{FF}^{2}+\beta_{FB}^{2}\rho_{FB}^{2})$ (ECU)	-5.947	43.688	-.136
For Brazil in the U.S.			
$-\beta_{0B}^{1}/(2\beta_{BB}^{1}+\beta_{BF}^{1}\rho_{BF}^{1})$ (intercept)	480.426	3075.250	.156
$-\beta_{BF}^{1}/(2\beta_{BB}^{1}+\beta_{BF}^{1}\rho_{BF}^{1})$ (Florida price)	.073	.584	.126
$(\beta_{BB}^{1}+\beta_{BF}^{1}\rho_{BF}^{1})/(2\beta_{BB}^{1}+\beta_{BF}^{1}\rho_{BF}^{1})$ (marg. cost)	-1.198	1.659	-.722
$-\beta_{B2}^{1}/(2\beta_{BB}^{1}+\beta_{BF}^{1}\rho_{BF}^{1})$ (income)	.211	.202	1.042
$-\beta_{B3}^{1}/(2\beta_{BB}^{1}+\beta_{BF}^{1}\rho_{BF}^{1})$ (Brazilian ER)	-.005	.005	-1.012
For Brazil in Europe			
$-\beta_{0B}^{2}/(2\beta_{BB}^{2}+\beta_{BF}^{2}\rho_{BF}^{2})$ (intercept)	732.240	4524.300	1.488
$-\beta_{BF}^{2}/(2\beta_{BB}^{2}+\beta_{BF}^{2}\rho_{BF}^{2})$ (Florida price)	.187	.612	.305
$(\beta_{BB}^{2}+\beta_{BF}^{2}\rho_{BF}^{2})/(2\beta_{BB}^{2}+\beta_{BF}^{2}\rho_{BF}^{2})$ (marg. cost)	.720	1.101	.654
$-\beta_{B2}^{2}/(2\beta_{BB}^{2}+\beta_{BF}^{2}\rho_{BF}^{2})$ (income)	-1.576	1.205	-1.308
$-\beta_{B3}^{2}/(2\beta_{BB}^{2}+\beta_{BF}^{2}\rho_{BF}^{2})$ (ECU)	-.464	.874	-.531

Table 5-6 Estimates of the price conjectural variations and initial inventory in the DDPG model.

	Parameter estimate	Standard error	t-statistic
For Florida in the U.S.			
ρ_{FB}^{1} (conjecture)	13.422	24.275	.553
β_{F6}^{1} (init. inventory)	-.103	.012	-8.616
For Florida in Europe			
ρ_{FB}^{2} (conjecture)	31.562	212.698	.148
β_{F6}^{2} (init. inventory)	-.001	.3E-03	-2.181
For Brazil in the U.S.			
ρ_{BF}^{1} (conjecture)	46.285	391.980	.118
β_{B6}^{1} (init. inventory)	-.099	.041	-2.431
For Brazil in Europe			
ρ_{BF}^{2} (conjecture)	-2.353	15.070	-.156
β_{B6}^{2} (init. inventory)	-.093	.040	-2.340

terms of sign consistency of the parameter estimates, the results are better than those in the DDQG model. Only the parameter estimates for income have the wrong sign. Some parameters, however, are not statistically significant. In the U.S. market, the own price coefficients are significant for both Florida and Brazil. Seasonality is also significant for the Brazil demand equation in the U.S. In Europe, only time and seasonal dummies are significant for both the Florida and Brazil demand functions.

Joint parameter test (Wald test) for each price reaction function indicates only the Florida price reaction function in Europe is not statistically significant. For the price reaction functions, only initial inventory (exogenous) is significant for both the Florida and Brazil reaction functions. The fact that the state variable inventory is important in pricing decisions may indicate that this variable *needs to be included* in the optimality conditions and/or some constraints on this variable should be imposed.[3] The results shown in Table 5-6 show that none of the price conjectures are statistically significant. This is rather surprising, since visual examination of the price data suggests that strong price conjectures are suspected. Tests of hypothesis for the Nash-in-price, consistent constant conjectures and Stackelberg-in-price conjectures in the DDPG model were also conducted. All of the hypotheses are rejected. The hypothesis of tacit/implicit collusive pricing in the U.S. market (t-dependent test) cannot be rejected (at five percent confidence level).

Some inconsistencies in the empirical results may be due to several possible reasons, such as inappropriate specification of the linear (inverse) demand functions. In addition, cost estimates used in this study may not be appropriate for the purposes of this study. This may also be due, in part, to the lack of appropriate data of exogenous variables for identification. The CIF price Tampa is used as the Brazilian FCOJ price for the U.S. market in this study. However, this official price may not be the appropriate price to use for those U.S. FCOJ imports which entered through other ports. Another reason may be inadequate information (of price variation). According to the available data, Brazilian FCOJ nominal prices (CIF Tampa) changed only ten times during the observed period. Brazilian FCOJ is needed to satisfy U.S. orange juice demand unfulfilled by Florida FCOJ supply. Hence, Brazilian FCOJ pricing/quantity decisions in the U.S. may depend not only on the Florida FCOJ situation, but also previous contractual arrangements and other important considerations which are excluded from the model. At the same time, U.S. FCOJ imports from other countries (such as Mexico) have tended to increase. Similarly, Florida FCOJ exports have tended to increase recently (both to Europe and ROW). Florida FCOJ exports to Europe, however, are still small compared to those of Brazil. In contrast, Florida FCOJ exports to ROW (such as Japan, Canada, and South Korea) may be relatively more important than its exports to Europe.

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[3] Among the possibilities that may result in this outcome are (1) nonlinear inventory cost, since *the maximum principle* for the costate variables yields the costate variables that now depend on the state variable (inventory level); (2) (inverse) demand functions that include the inventory level as exogenous variables, since *the maximum principle* for the control variables *(i.e.*, maximizing the current value Hamiltonians with respect to the control variable) will result in the control variables that depend on the state variables/inventory (*i.e.*, the control variables now have a feedback form).

CHAPTER 6
SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE RESEARCH

Summary

Agricultural economists have increasingly acknowledged that some forms of imperfect competition exist in domestic and international agricultural and processed food markets. It is widely argued that interdependence among market participants makes it important to view imperfect competition as a dynamic process. The general objective of this study was to investigate short-run dynamic competition in the world orange juice market. The most established approach to industrial organization (IO), the *structure-conduct-performance* (SCP) paradigm, provides rich and flexible methodology for this study. Models developed in this study assume a duopoly market structure which consists of the two major producers (which together account for approximately 85%) of orange juice in the world: Brazil and Florida. Based on the profit-maximizing behavior assumption, optimal strategies for each producer in the short-run dynamic duopoly game models were derived. Theoretical models were developed to capture the dynamic elements in the competition due to the market structure of imperfect competition and to spatial and intertemporal optimization.

Most previous studies of the world orange juice market were based on econometric models and focused on the long-term horizon. Financial viability, seasonal variation in production, relatively slow growth and inelastic demand, and the nature of the market structure are among reasons why short-run decisions are at least equally important as long-run planning. In the short run, the orange juice industry is restricted by past investment decisions. Production in a particular year, for example, is the result of planting decisions in previous years. Therefore, short-run strategies have to take into account this aspect and allow adjustment. In this study, supplies are treated as exogenous to the system.

Two general models were developed: the dynamic duopoly quantity game (DDQG), and the dynamic duopoly price game (DDPG) models. Both models were developed as noncooperative differential games. The U.S. and European markets account for approximately 86% of world orange juice consumption. Hence, even though supply and demand in the rest of the world may be increasingly important in the future, they are assumed to not be determining factors in this study. Differentiated products were assumed in the models. The study also assumed linear and market-specific (selling) cost functions and linear inventory-related costs for both players.

Analytical solutions were derived and investigated. Nash and Stackelberg strategies were derived, for both differentiated and homogeneous products. Some results on tacit collusion were also investigated. For specific (closed-form) solutions, a linear demand function was investigated in the models. The theoretical results provide foundations for the empirical analysis conducted in this study. Linear inverse demand functions were estimated for the empirical analysis in the quantity game (DDQG) model. Constant conjectural variation parameters were assumed and estimated. The observed and simulated optimal results were compared. A simultaneous equation system of linear product demand and price reaction functions derived from the optimal solution was established in the empirical analysis with the price game (DDPG) model. Thus, price elasticities of demand were estimated along with the price conjectural variations and price reaction function. Pricing strategies of Nash, Stackelberg, collusive, and competitive were hypothesized as the pricing behavior of the players.

Conclusions

The key implication of this study, from a methodological perspective, relates to the importance of ensuring that the crucial dimensions of a short-run dynamic profit maximization objective (embeded in the structure-conduct relation, such as: the structure of competition, product types, quantity decisions, pricing, inventory strategies, relevant time horizon, and spatial dimension) are adequately captured in the model. It is believed that marketing decisions, *i.e.*, quantity and pricing decisions, spatial and inventory decisions are important in short-run optimization.

Since the profit functions in the two models are differentiable and jointly concave in the control and state variables, and the equations of motion are linear in the control and state variables, the necessary optimality conditions are also sufficient. It was shown that due to the structure of the two models developed here, the games are *state separable*, which means that for each player, neither the optimal quantity in the DDQG model (optimal price in the DDPG model) nor the costate/adjoint equations depend on the state variable *I* (the level of inventory). Consequently, in both models, the optimal open-loop solutions coincide with the optimal feedback solutions.

The study also shows that, theoretically, noncooperative behavior among players in a duopoly competition, tacit/implicit price collusions are among possible outcomes resulting from optimizing behavior, whether in the DDQG or DDPG Models.

In the empirical analysis, the profit maximizing objective was pursued through an integrated spatial and intertemporal pricing and inventory strategy employed by both Florida and Brazilian FCOJ processors across a short-run (a season) time horizon.

The empirical results indicate that the price game model provides slightly better estimates than those obtained from the quantity game model. Some parameter estimates, however, are not found to be statistically significant. Except for the hypothesis of consistent constant conjecture for Brazil in the U.S. market for the quantity game model, all of the hypotheses of the Nash, Stackelberg, constant consistent conjectures are rejected, both for the quantity and price games. The hypothesis of tacit/implicit collusive price in the U.S. market cannot be rejected.

Limitations and Suggestions for Future Research

It is widely acknowledged that as a process of inquiry becomes more scientific, it is more likely to obtain *correct* answers to questions and *better* solutions. This does not necessarily mean that better results always occur when scientific approaches are used. Needless to say, even when a scientific approach leads to better solutions, for some reason it is not always preferred in actual practice. The models discussed in the previous section provides a framework to compare the optimal policies derived from the real world behavior of the frozen concentrate orange juice markets in the United States and Europe. The bargaining game of discovering the balance between mathematical (and statistical) convenience and realism in economic studies sometimes (or perhaps usually) leads to uncertainties or questionable robustness of the results and/or conclusions. This study has been no exception.

The empirical results demonstrate that the price game model performs better than the quantity game model. It should be noted, however, that this study is conducted at the aggregate industry level. Had the analysis been conducted at the firm level, the conclusions would not necessarily be the same. Another caveat to this study is that the models are deterministic. Hence, expectations/speculation do not play a role, while in reality these may be important due to seasonality and the uncertainty of production. These, along with a consideration of the freeze factor, may be incorporated into stochastic models as an extention to the deterministic models.

Another limitation is the assumption of two players and two markets in the international FCOJ market. Markets outside of the U.S. and Europe are growing. Under NAFTA, it is possible that Mexico will become a more important supplier of FCOJ. Thus, an extention to oligopoly model will be more appropriate in the future. It is assumed that the players maximize their individual profit functions along a fixed short-run planning horizon. They ignore the role of other parties, especially with respect to the vertical market system (input markets and consumers). A possibility of a shift in market power in the vertical system is another important factor that may be considered. Retailers may now hold more power than before, and thus oligopsony may be considered as an alternative appropriate model for the FCOJ market in the future.

Single product decision is also assumed in the model. In reality, citrus is produced and marketed in various forms which may be interrelated. Therefore, models for joint-product decisions are important to capture a more complete analysis of the market. The models also assume market independence. The U.S. and Europe are still the most important markets, and thus decisions related to one market are likely affect those to the other. Growing cooperation among countries such as NAFTA and EEC are also important issues to address.

One may argue that rivalry is generally expensive to the players. Therefore, there is always enticement for the participants in duopoly/oligopoly competition to collude, and thus avoid detrimental effects of severe competition such as price wars. Players then may prefer non-price competition and chooses other strategies such as advertising.

The study assumed that each player only has one strategic instrument to choose. In addition, they are assumed to be identical strategic instruments, *i.e.*, both players choose quantity in the DDQG model and price in the DDPG model. Technological and/or geographical differences may cause a player not to employ the same strategic instrument with that of the rival. Therefore, unidentical strategy game models are among possible extentions to the models developed in this study.

Even though there are two markets considered in the models, since they are assumed to be independent, the models are not truly spatial optimization models, in the sense that decisions in a market do not directly affect the decision in the other market. They are indirectly related through the corresponding equations of motion/the rates of change of inventory. Yet, since the inventory related costs are assumed linear, and no particular inventory policy restrictions are imposed, the effect of inventory on the optimal decisions is not as crucial had the costs been assumed highly nonlinear and some inventory restrictions been imposed along the time horizon. These are typical dilemmas in spatial and dynamic optimization. Even for simple problems, the analytical procedures are generally lengthy and tedious. Increasing the dimension of the models should be undertaken with caution, however, since they will generally be extremely complex. Even if they are solvable, this does not necessarily mean that complex models are operational for empirical analyses and/or able to better capture real world problems.

APPENDIX A
A STATIC ANALYSIS OF DUOPOLY GAMES

Quantity Game

To derive static duopoly games, I will start with a quantity game in a differentiated duopoly. Suppose the individual inverse demand function is defined by

$$p_i = p_i(u_i,\ u_j), \tag{A-1}$$

where

(*i*) $\dfrac{\partial p_i}{\partial u_i} \le 0, \quad and \quad < 0 \quad if \quad u_i > 0,$

(*ii*) $\dfrac{\partial p_i}{\partial u_j} \ge 0, \quad and \quad > 0 \quad if \quad u_j > 0; \quad and$

(*iii*) $\dfrac{\partial p_i}{\partial u_i}.\dfrac{\partial p_j}{\partial u_j} - \dfrac{\partial p_i}{\partial u_j}.\dfrac{\partial p_j}{\partial u_i} > 0, \quad (i \neq j).$

Further assume that marginal cost (MC_i) of each player is constant, and that the profit maximization for player *i* is

$$\Pi_i = (p_i - MC_i)u_i, \tag{A-2}$$

where

$$(i)\ \frac{\partial^2 \Pi_i}{\partial u_i^2} \leq 0, \quad \textit{and} \quad < 0 \ \textit{ if } \ u_i > 0,$$

$$(ii)\ \frac{\partial^2 \Pi_i}{\partial u_i \partial u_j} \leq 0, \quad \textit{and} \quad < 0 \ \textit{ if } \ u_j > 0; \ \textit{and}$$

$$(iii)\ \frac{\partial^2 \Pi_i}{\partial u_i^2} \cdot \frac{\partial^2 \Pi_j}{\partial u_j^2} - \frac{\partial^2 \Pi_i}{\partial u_i \partial u_j} \cdot \frac{\partial^2 \Pi_j}{\partial u_i \partial u_j} > 0, \quad (i \neq j).$$

It follows that

$$p_i = \frac{MC_i}{1 + \dfrac{1}{\eta_{ii}} + \dfrac{\gamma_{ij}}{\eta_{ij}} \dfrac{u_i}{u_j}}. \tag{A-3}$$

where $1/\eta_{ii}$, $1/\eta_{ij}$, and γ_{ii} are own-price flexibilities of demand, cross-price flexibilities of demand, and the conjectural variations of player *i* about player *j*.

To derive the classical Nash-Cournot and Stackelberg strategies in this static quantity game, I will specify a linear inverse demand function

$$p_i = \alpha_{0i} + \alpha_{ii}u_i + \alpha_{ij}u_j, \tag{A-4}$$

where own-price flexibilities of demand, $1/\eta_{ii}$, and cross-price flexibilities of demand, η_{ij}, are

$$\eta_{ii} = \frac{p_i}{\alpha_{ii} u_i}, \qquad \mathit{and} \qquad \eta_{ij} = \frac{p_i}{\alpha_{ij} u_j} \tag{A-5}$$

The optimality condition for player *i* gives the reaction function

$$u_i = \frac{MC_i - \alpha_{0i} - \alpha_{ij} u_j}{2\alpha_{ii} + \alpha_{ij}\gamma_{ij}} \tag{A-6}$$

Rearrange (A-6) for player *i*'s CV

$$\gamma_{ij} = \frac{MC_i - (\alpha_{0i} + 2\alpha_{ii} u_i + \alpha_{ij} u_j)}{\alpha_{ij} u_i} \tag{A-7}$$

The Nash-Cournot strategy for player *i* are (Nash-Cournot strategy for player *j* are identical):

$$u_i = \frac{2\alpha_{jj}(MC_i - \alpha_{0i}) - \alpha_{ij}(MC_j - \alpha_{0j})}{4\alpha_{ii}\alpha_{jj} - \alpha_{ij}\alpha_{ji}} \tag{A-8}$$

To derive the Stackelberg strategies (assuming player *i* is the leader), player *i*'s CV needs to be defined, which can be derived from *j*'s reaction function (A-6) (modified for *j*)

$$u_j = \frac{MC_j - \alpha_{0j} - \alpha_{ji}u_i}{2\alpha_{jj}} \tag{A-9}$$

and since marginal cost MC_i is constant, the player *i*'s CV about player *j* is

$$\gamma_{ij} = \frac{-\alpha_{ji}}{2\alpha_{jj}}. \tag{A-10}$$

Equations (A-6), (A-10), and (A-11) gives the Stackelberg strategy for player *i* (the leader) as

$$u_i = \frac{2\alpha_{jj}(MC_i - \alpha_{0i}) - \alpha_{ij}(MC_j - \alpha_{0j})}{2(2\alpha_{ii}\alpha_{jj} - \alpha_{ij}\alpha_{ji})}. \tag{A-11}$$

The Stackelberg strategy for player *j* (the follower) is

$$u_j = \frac{(4\alpha_{ii}\alpha_{JJ} - \alpha_{ij}\alpha_{ji})(MC_j - \alpha_{0j}) - 2\alpha_{ij}\alpha_{ji}(MC_i - \alpha_{0i})}{4\alpha_{jj}(2\alpha_{ii}\alpha_{jj} - \alpha_{ij}\alpha_{ji})}. \tag{A-12}$$

Nash-Cournot or Stackelberg strategies may be derived as a special case where products are assumed homogeneous ((A-4) becomes $p = \alpha_0 + \alpha_1 (u_i + u_j)$). Nash-Cournot is obtained from (A-8) yields

$$u_i = \frac{2MC_i - MC_j - \alpha_0}{3\alpha_1},$$

$$\Pi_i = \frac{1}{9\alpha_1}(2(MC_i^2 + MC_j^2) - 5MC_i MC_j + \alpha_0(MC_i + MC_j - \alpha_0));$$

$$p = \frac{1}{3}(\alpha_0 + MC_i + MC_j). \qquad \text{(A-13)}$$

In the trivial case of zero marginal cost, $MC_i = MC_j = 0$, we get

$$u_i = u_j = -\frac{\alpha_0}{3\alpha_1}; \qquad p = \frac{\alpha_0}{3} \quad \textit{and}$$

$$\Pi_i = \Pi_j = -\frac{\alpha_0^2}{9\alpha_1} \qquad \text{(A-14)}$$

Compare to perfect competition and monopoly cases:

Competitive case

For a perfect competitive market, where each firm/player has identical constant marginal cost MC_i, the equilibrium is

$$u_i = \frac{MC_i - \alpha_0}{\alpha_1}; \qquad p = MC_i, \quad \text{and} \qquad \text{(A-15)}$$

$$\Pi_i = 0;$$

Monopoly Case

In a monopoly case, the market equilibrium from a profit maximization is

$$u = \frac{1}{2\alpha_1}(MC - \alpha_0); \qquad p = \frac{1}{2}(\alpha_0 + MC), \quad \text{and}$$

$$\Pi = -\frac{1}{4\alpha_1}(MC - \alpha_0)^2 \qquad \text{(A-16)}$$

Stackelberg strategies, with player *i* is the leader and *j* is the follower, yield

$$u_i = \frac{2MC_i - MC_j - \alpha_0}{2\alpha_1},$$

$$\Pi_i = -\frac{1}{8\alpha_1}(2MC_i - MC_j - \alpha_0)^2,$$

$$u_j = \frac{3MC_j - 2MC_i - \alpha_0}{4\alpha_1}, \qquad \text{(A-17)}$$

$$\Pi_j = -\frac{1}{16\alpha_1}(2MC_i - 3MC_j + \alpha_0)^2, \quad \text{and}$$

$$p = \frac{1}{4}(2MC_i + MC_j + \alpha_0).$$

The above analysis shows that in a Cournot (quantity) duopoly game, the total industry output of the Stackelberg strategy is lower than that of the Nash strategy. Consequently, the market price of the Stackelberg strategy is higher than that of the Nash strategy. The Stackelberg strategy also shows that the profit of the leader is higher than that of the follower. Because there is an incentive for potential gain from being the leader for each player in this quantity game, it is likely that each will make decision based upon the wrong assumption about the rival's behavior. Many authors call this phenomenon as the *Stackelberg disequilibrium*.

Price Game

The main problem with the original Bertrand model is that, unless the marginal costs of the duopolists are identical, without any further assumptions Bertrand (*pure* price) competition will have no stable equilibrium/solution or a solution where one player (the higher cost participant) is expelled from the market. To show this, I assume a linear demand function

$$u_i = \beta_{0i} + \beta_{ii} \, p_i + \beta_{ij} \, p_j. \tag{A-18}$$

Profit maximization with respect to the price p_i give the reacttion function for player i (and for j is identical)

$$p_i = \frac{(\beta_{ii} + \beta_{ij}\rho_{ij}) \, MC_i - \beta_{0i} - \beta_{ij} \, p_j}{2\beta_{ii} + \beta_{ij}\rho_{ij}}, \tag{A-19}$$

where ρ_{ij} is the price conjectural variations of player *i* about player *j*.

Nash-Bertrand conjectures result in

$$p_i = \frac{2\beta_{ii}\beta_{jj}\ MC_i - \beta_{ij}\beta_{ji}\ MC_j - 2\beta_{0i}\beta_{jj} + \beta_{0j}\beta_{ij}}{4\beta_{ii}\beta_{jj} - \beta_{ij}\beta_{ji}}, \qquad \text{(A-20)}$$

If we assume homogeneous product then (A-20) yields

$$p_i = \frac{\beta_1\ (2MC_i - MC_j) - \beta_0}{3\beta_1},$$

$$p_j = \frac{\beta_1\ (2MC_j - MC_i) - \beta_0}{3\beta_1}, \quad \text{and} \qquad \text{(A-21)}$$

$$p_i = p_j = \bar{p}.$$

Thus, unless the marginal cost are equal, without additional assumptions, one player will be forced to exit from the market. For an identical cost case ($MC_i = MC_j = MC$), we obtain an indeterminate solution where

$$u_i \geq 0; \quad u_j \geq 0; \quad \text{and} \quad u_i + u_j = \frac{1}{3}(2\beta_0 - \beta_1 MC). \qquad \text{(A-21)}$$

(Intuitively, we will expect that $u_i = u_j$, since the costs are identical).

Suppose now that player i is the price leader and player j is the follower. Player's i price conjecture can be derived from player j's reaction function (similar to (A-19), with $\rho_{ji} = 0$), which is

$$\rho_{ij} = \frac{\partial p_j}{\partial p_i} = \frac{-\beta_{ji}}{\beta_{jj}} \tag{A-22}$$

Substituting (A-22) into (A-19) altogether with player j's recation equation will yield the solutions similar to (4-43) (or (4-46) in collusive price case). When products are homogeneous and prices collude then it yields

$$p = \frac{2\beta_1 c_i - 3\beta_0}{2\beta_1} = \frac{2\beta_1 c_j - \beta_0}{2\beta_1}, \quad or \\ \beta_1(c_j - c_i) + \beta_0 = 0. \tag{A-23}$$

Hence, collusive price leadership is possible only if the price leader is more efficient than the price follower.

APPENDIX B
A DYNAMIC ANALYSIS OF AN INVENTORY PROBLEM

Consider a firm that has to determine an optimal inventory policy given that the production rate at every period is fixed. The optimal control problem is then as follows.

Competitive Case

Suppose the firm is in a competitive market with a linear industry demand curve of the type $u(t) = a - (a/b)\, p(t)$, has a constant marginal cost u, and has to decide its optimal sale decisions. Its optimal control problem is then

$$Maximize_u \Pi = \int_0^T e^{-\delta t}[(p(t) - c)u(t) - hI(t)]dt + e^{-\delta T}SI(T) \qquad \text{(B-1)}$$

Subject to

$$\dot{I}(t) = q(t) - u(t) \qquad \text{(B-2)}$$

The current Hamiltonian for this problem is

$$H = (p - c)u - hI + \lambda(q - u), \qquad \text{(B-3)}$$

and its first order necessary conditions are

$$
\begin{aligned}
&(i)\ \ H_u = p(t) - c - \lambda(t) = 0,\\
&(ii)\ \ \dot{\lambda}(t) = \delta\lambda(t) - h, \qquad \textit{and}\ \ \lambda(T) = S, \\
&(iii)\ \ \dot{I}(t) = q(t) - u(t).
\end{aligned}
\tag{B-4}
$$

From *(ii)*, we get

$$
\lambda(t) = \frac{1}{\delta}(h + (\delta S - h)e^{\delta(t - T)}),
\tag{B-5}
$$

and substituting it to *(i)*, we have

$$
p(t) = c + \frac{1}{\delta}(h + (\delta S - h)e^{\delta(t - T)}).
\tag{B-6}
$$

Monopoly Case

If the firm is a monopolist, its first order necessary conditions are then

$$
\begin{aligned}
&(i)\ \ H_p = [p(t) + u(t)\frac{\partial p(t)}{\partial u(t)}] - c - \lambda(t) = 0,\ or\\
&\quad\ MR = c + \lambda(t),\\
&(ii)\ \ \dot{\lambda}(t) = \delta\lambda(t) - h, \qquad \textit{and}\ \ \lambda(T) = S, \\
&(iii)\ \ \dot{I}(t) = q(t) - u(t).
\end{aligned}
\tag{B-7}
$$

As we can see, in both cases, the static solution (Appendix A, equation A-1) is the limiting case of the dynamic solution as the discount rate $\delta \to \infty$.

APPENDIX C
RELATIONSHIP POSSIBILITIES BETWEEN TWO DIFFERENT GAMES

Krouse (1990) shows possible relationship of conjectural variations resulting from different strategic games (see also Kamien and Schwartz, 1983). Suppose that each player (i and j) in a duopoly competition selects p (p_i and p_j, respectively), as the strategic variable. Assume that a unique equilibrium exists such that

$$\frac{dp_j}{dp_i} = \rho_{ij}(p) \neq 0. \tag{C-1}$$

Suppose that we are able to specify new different strategies u_i, where $u_i = f_i(p)$ are well-defined and suitably regular functions. We may derive

$$\frac{du_i}{dp_i} = \frac{\partial f_i}{\partial p_i} + \frac{\partial p_j}{\partial p_i}. \tag{C-2}$$

The ratio between (C-2) and (C-1) may be written as

$$\frac{du_i}{dp_j} = \frac{\left(\frac{\partial f_i}{\partial p_i} + \frac{\partial f_i}{\partial p_j}\frac{dp_j}{dp_i}\right)}{\rho_{ij}}. \tag{C-3}$$

or equivalently

$$\frac{du_i}{du_j}\,\frac{du_j}{dp_j} = \frac{\left(\dfrac{\partial f_i}{\partial p_i} + \dfrac{\partial f_i}{\partial p_j}\,\dfrac{dp_j}{dp_i}\right)}{\rho_{ij}}. \tag{C-4}$$

If the conjectural variation in the new strategic variables are Nash, i. e., $\gamma_{ji} = 0$, then (C-4) provides the corresponding conjectural variation in p (i. e., ρ_{ij})

$$\frac{dp_j}{dp_i} = \rho_{ij}(p) = -\,\frac{(\partial f_i \,/\, \partial p_i)}{(\partial f_i \,/\, \partial p_j)}. \tag{C-5}$$

Thus, this shows that any Nash conjecture in a particular strategy may correspond to a non-Nash conjecture in a different strategy.

For the DDQG and the DDPG models in this study, with non-zero prices and quantities and assuming that the shadow value term is negligible, it turns out that the relationship for player i (and similarly for player j) is

$$\gamma_{ij}^{k} = \frac{1 - \alpha_{ii}^{k}(\beta_{ii}^{k} + \beta_{ij}^{k}\rho_{ij}^{k})}{\alpha_{ij}^{k}(\beta_{ii}^{k} + \beta_{ij}^{k}\rho_{ij}^{k})}. \tag{C-6}$$

APPENDIX D
DATA

Table D-1 Florida FCOJ data (thousand SSE gallons) July 1990 - June 1993 (Source: FDOC).

Year: Month	Productio	Initial Inventory	End Inventory	Movemen to the US	Exports t Europe	Exports t ROW
90:7	14220	356139	316941	44151	2423	6845
90:8	16516	316941	274300	48673	1491	8993
90:9	33263	274300	232784	66375	1375	7030
90:10	21306	232784	198392	49890	809	4997
90:11	33761	198392	161526	63684	1203	5740
90:12	255796	161526	349211	59240	2251	6620
91:1	120270	349211	400853	60906	1329	6394
91:2	68718	400853	402244	60328	1221	5778
91:3	116196	402244	429439	82892	1468	4641
91:4	100133	429439	462011	58220	2021	7320
91:5	60782	462011	458353	56141	2284	6015
91:6	10142	458353	385683	73182	2463	7167
91:7	15482	385683	336761	54498	2099	7807
91:8	22043	336761	274812	77446	1758	4788
91:9	26570	274812	237058	56597	1403	6324
91:10	14481	237058	177811	65124	2248	6357
91:11	40564	177811	128569	83132	1795	4879
91:12	192937	128569	248510	63668	1988	7340
92:1	153836	248510	340865	54492	2098	4891
92:2	79135	340865	347065	65670	1980	5285
92:3	53296	347065	336894	55209	1715	6543
92:4	102694	336894	376788	52493	2086	8221
92:5	98990	376788	399455	66161	1901	8260
92:6	16617	399455	350852	53023	4077	8120
92:7	10029	350852	302364	48117	2295	8105
92:8	20093	302364	245259	68664	3482	5053
92:9	21970	245259	203374	55388	2446	6022
92:10	51571	203374	175254	71616	3244	4831
92:11	13610	175254	125014	56182	1992	5676
92:12	176422	125014	231913	62754	1801	4968
93:1	205211	231913	347611	82570	2314	4628
93:2	179624	347611	443268	75393	2385	6190
93:3	81279	443268	430117	82446	4146	7839
93:4	117400	430117	457229	77772	5892	6623
93:5	139410	457229	496679	88221	4815	6925
93:6	69239	496679	475953	78844	3717	7404

Table D-2 Brazilian FCOJ data (thousand SSE gallons) July 1990 - June 1993 (Source: FDOC) .

Year: Month	Production	Intial Inventory	End Inventory	Exports to the US	Exports to Europe	Exports to ROW
90:7	89611	110	15224	34371	32473	7653
90:8	125646	15224	47916	34452	48426	10076
90:9	121418	47916	90839	21770	44508	12217
90:10	101581	90839	92882	54807	32634	12097
90:11	104606	92882	61197	62966	59210	14115
90:12	115727	61197	32191	37683	83726	23324
91:1	135501	32191	79948	24031	31665	32048
91:2	85919	79948	90845	18821	36431	19770
91:3	92712	90845	76871	22658	60179	23849
91:4	77269	76871	56460	27909	46689	23082
91:5	59461	56460	48896	9532	31301	26192
91:6	92350	48896	58429	21648	51051	10118
91:7	98544	58429	61789	47578	35906	11700
91:8	138171	61789	76867	40146	73548	9399
91:9	133521	76867	104905	16519	75810	13154
91:10	111707	104905	34027	75890	90713	15982
91:11	115033	34027	-5495	54969	78637	20949
91:12	127263	-5495	-6454	69509	28353	30360
92:1	149008	-6454	59365	13046	60381	9762
92:2	94484	59365	35643	38490	62804	16912
92:3	101954	35643	70200	38814	17650	10933
92:4	84972	70200	54414	24879	44508	31371
92:5	65389	54414	29304	23022	57110	10367
92:6	101556	29304	46662	8037	46245	29916
92:7	118892	46662	63706	20356	69226	12266
92:8	166702	63706	111597	58160	49597	11054
92:9	161092	111597	125242	55615	80212	11620
92:10	134774	125242	119435	57231	65793	17557
92:11	138787	119435	128506	46447	69832	13437
92:12	153542	128506	126604	58725	72417	24302
93:1	179777	126604	174201	44630	75931	11619
93:2	113994	174201	181581	19589	64743	22282
93:3	123006	181581	181777	8764	75204	38842
93:4	102518	181777	168957	9168	88249	17921
93:5	78891	168957	90425	20558	111877	24988
93:6	122526	90425	158802	14580	26010	13559

Note: Production and inventory data are estimated.

Table D-3 Florida FCOJ price (deflated) and marginal cost data (US $ per thousand SSE gallons) July 1990 - June 1993 (Source: FDOC) .

Year: Month	Price in the US	Price in Europe	Mar. cost to the US	Mar. cost to Europe
90:7	1566.639	1525.295	1313.584	1648.881
90:8	1568.811	1715.288	1304.749	1638.367
90:9	1370.093	1880.997	1295.645	1627.533
90:10	1375.818	1783.684	1286.264	1616.37
90:11	1038.505	1287.344	1276.597	1604.866
90:12	936.1328	1446.957	1266.636	1593.013
91:1	938.1812	1603.247	1256.372	1580.798
91:2	951.3657	1472.811	1245.795	1568.212
91:3	959.8854	1237.705	1234.895	1555.241
91:4	960.6023	1285.709	1223.664	1541.876
91:5	964.2027	1224.458	1212.091	1528.104
91:6	974.4291	1334.035	1200.165	1513.912
91:7	980.3708	1105.546	1337.084	1676.846
91:8	979.6241	1284.733	1328.249	1666.332
91:9	975.9078	1350.271	1319.145	1655.498
91:10	983.3688	1255.938	1309.764	1644.335
91:11	1210.126	1291.53	1300.097	1632.831
91:12	1366.271	1252.366	1290.136	1620.978
92:1	1312.496	1527.464	1279.872	1608.763
92:2	1284.412	1308.459	1269.295	1596.177
92:3	1280.757	1510.853	1258.395	1583.206
92:4	1268.13	1398.198	1247.164	1569.841
92:5	1145.747	1486.992	1235.591	1556.069
92:6	1103.285	1332.408	1223.665	1541.877
92:7	1105.656	1272.56	1325.099	1662.584
92:8	982.4596	1284.161	1316.264	1652.07
92:9	983.1654	1342.947	1307.16	1641.236
92:10	996.7707	1316.256	1297.779	1630.073
92:11	890.1881	1242.255	1288.112	1618.569
92:12	821.3724	1214.478	1278.151	1606.715
93:1	733.8944	1423.808	1267.887	1594.501
93:2	665.5967	1236.148	1257.31	1581.915
93:3	680.7814	1213.604	1246.41	1568.944
93:4	743.382	950.881	1235.179	1555.579
93:5	755.4461	1172.384	1223.606	1541.807
93:6	884.2944	1162.106	1211.68	1527.615

Note: Marginal cost (including the shadow values of inventory) are estimated based on Muraro (1993).

Table D-4 Brazilian FCOJ price (deflated) and marginal cost data
(US $ per thousand SSE gallons) July 1990 - June 1993
(Source: FDOC and Foodnews) .

Year: Month	Price in the US	Price in Europe	Mar. cost to the US	Mar. cost to Europe
90:7	1509.675	1652.302	1292.388	1121.608
90:8	1340.624	1508.834	1283.548	1111.088
90:9	1341.553	1364.785	1274.448	1100.259
90:10	967.3747	1236.023	1265.068	1089.097
90:11	987.3134	1003.882	1255.398	1077.59
90:12	1011.026	1003.083	1245.438	1065.737
91:1	1013.239	997.5279	1235.178	1053.528
91:2	1027.478	993.5975	1224.598	1040.938
91:3	1036.679	992.034	1213.698	1027.967
91:4	1037.453	985.829	1202.468	1014.603
91:5	1041.342	982.7555	1190.898	1000.835
91:6	1052.386	978.9404	1178.968	986.6382
91:7	1058.803	974.4013	1292.388	1121.608
91:8	1057.997	831.4419	1283.548	1111.088
91:9	1053.983	859.4446	1274.448	1100.259
91:10	1219.381	1077.303	1265.068	1089.097
91:11	1366.275	1199.463	1255.398	1077.59
91:12	1366.275	1197.64	1245.438	1065.737
92:1	1312.5	1193.107	1235.178	1053.528
92:2	1284.415	1187.712	1224.598	1040.938
92:3	1280.761	1182.366	1213.698	1027.967
92:4	1268.134	1176.19	1202.468	1014.603
92:5	1087.738	1171.817	1190.898	1000.835
92:6	1103.288	1016.875	1178.968	986.6382
92:7	1105.659	1016.875	1292.388	1121.608
92:8	1108.04	814.7652	1283.548	1111.088
92:9	1108.836	811.1574	1274.448	1100.259
92:10	1124.181	716.6836	1265.068	1089.097
92:11	890.1906	669.4611	1255.398	1077.59
92:12	897.4279	639.0311	1245.438	1065.737
93:1	733.8964	606.8202	1235.178	1053.528
93:2	726.803	573.96	1224.598	1040.938
93:3	743.384	571.4627	1213.698	1027.967
93:4	743.384	568.987	1202.468	1014.603
93:5	755.4482	567.3485	1190.898	1000.835
93:6	948.6094	700.7115	1178.968	986.6382

Note: Marginal cost (including the shadow values of inventory) are estimated based on Muraro (1993).

Table D-5 Data of exogenous variables (July 1990 - June 1993)

Year: Month	Income in West Germany	European currency Unit	Brazilian exchange rate	Brazilian season.	Income in the US	Apple J. cons. in the US	Price of AJ in the US	Florida season in the US	Florida season. in Europe	Time dummy [year]
90:7	3273.365	1263.5	68.37	1	11644.61	16878.49	2013.841	0	1	1
90:8	3347.859	1316.8	71.5	1	11584.17	16899.08	2037.422	0	0	1
90:9	3422.178	1317.7	84.1	1	11530.67	18221.27	2018.031	0	0	1
90:10	3398.039	1360	110.3	1	11428.87	19505.63	1984.68	0	0	1
90:11	3469.932	1380.8	141.7	1	11450.08	16767.73	2132.196	0	0	1
90:12	3404.068	1360.1	170.35	1	11494.43	17091.6	2227.074	0	0	1
91:1	3387.88	1363.9	220.1	0	11770.12	18626.51	2217.36	1	0	1
91:2	3321.349	1391.9	221	0	11751.2	16791.19	2322.485	1	0	1
91:3	3378.22	1279.2	239.2	0	11817.26	17643.67	2402.985	1	1	1
91:4	3384.804	1210.1	260.73	0	11790.51	16450.56	2427.184	1	1	1
91:5	3440.585	1198.9	284.7	0	11834.81	16397.02	2481.259	1	1	1
91:6	3451.685	1151.3	312.75	0	11857.97	14877.78	2545.455	0	1	1
91:7	3427.591	1150.1	329.47	1	11802.15	14902.84	2599.085	0	1	2
91:8	3438.528	1176.4	394.75	1	11811.41	14722.79	2658.035	0	0	2
91:9	3502.728	1209.3	535	1	11804.6	15503.57	2647.951	0	0	2
91:10	3443.096	1211.4	647.1	1	11802.17	16513.13	2614.679	0	0	2
91:11	3504.785	1257.9	840.4	1	11687.86	15393.82	2663.126	0	0	2
91:12	3448.492	1300.4	1057.2	1	11807.21	15940.44	2708.649	1	0	2
92:1	3439.176	1294.6	1319.2	0	11778.18	16290.71	2631.195	1	0	2

Table D-5 Data of exogenous variables (July 1990 - June 1993) (continued)

92:2	3382.367	1262.9	1630.9	0	11825.94	15615.33	2603.424	1	0	2
92:3	3405.901	1230.4	1988	0	11864.55	15974.25	2638.691	1	1	2
92:4	3395.874	1242.4	2396.1	0	11824.91	14744.85	2640.845	1	1	2
92:5	3419.071	1268.1	2849.2	0	11833.37	14444.01	2670.895	1	1	2
92:6	3442.215	1303.2	3446.8	0	11815.01	13083.13	2694.782	0	1	2
92:7	3417.441	1370.5	4204.6	1	11798.57	13438.03	2693.41	0	1	3
92:8	3440.161	1401.9	5131	1	11820.81	13540.58	2713.568	0	0	3
92:9	3491.535	1384	6400	1	11802.73	14389.62	2679.598	0	0	3
92:10	3437.014	1325.5	8034.1	1	12073.28	15942.9	2600.146	0	0	3
92:11	3486.628	1239.2	9949.9	1	12029.48	14791.43	2675.953	0	0	3
92:12	3378.284	1239	12243	1	12583.7	15316.3	2653.363	1	0	3
93:1	3327.655	1211.5	15720	0	11918.24	16733.58	2620.12	1	0	3
93:2	3217.906	1182.3	19858	0	11924.85	16089.26	2513.011	1	0	3
93:3	3259.043	1178.6	25122	0	11993.99	17249.9	2608.365	1	1	3
93:4	3266.352	1221.1	32268	0	12094.04	16066.48	2593.156	1	1	3
93:5	3306.886	1217.2	41700	0	12097.87	16425.09	2619.784	1	1	3
93:6	3347.329	1184.3	54336.7	0	12062.59	15010.94	2632.813	0	1	3

Source: - West Germany GDP (constant price 1991) : OECD
- European currency unit : IMF
- Brazilian exchange rate : CACEX/Banco do Brasil.
- Disposable income in the US (constant price 1982) : US Dept. of Commerce
- Apple juice : Nielsen
- Population : United Nations.

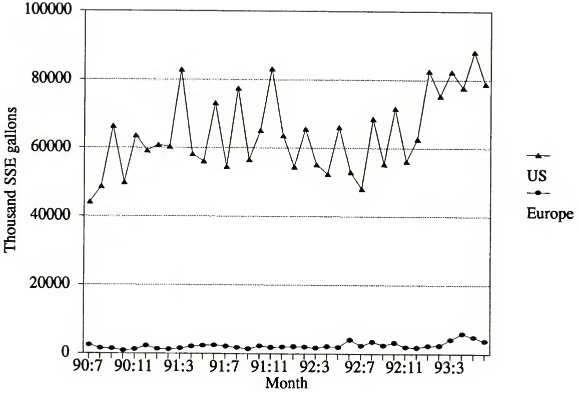

Figure D-1
Florida FCOJ movement

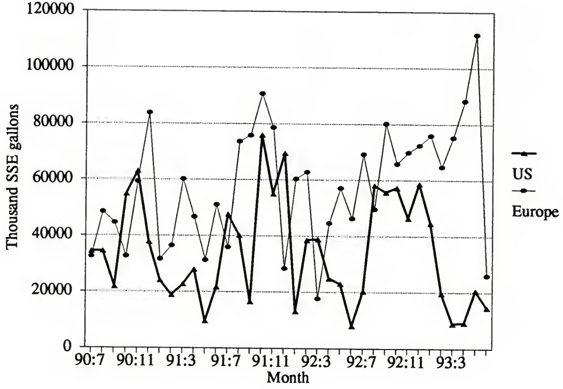

Figure D-2
Brazilian FCOJ movement

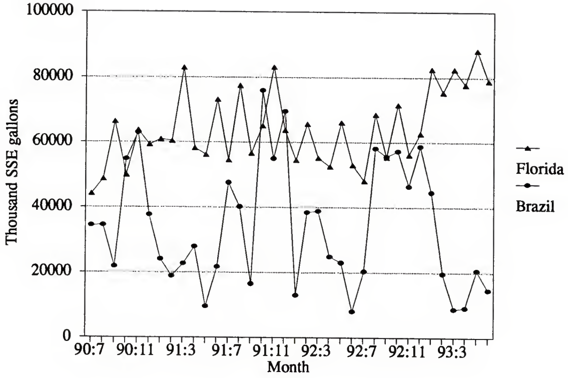

Figure D-3
FCOJ movement in the US market.

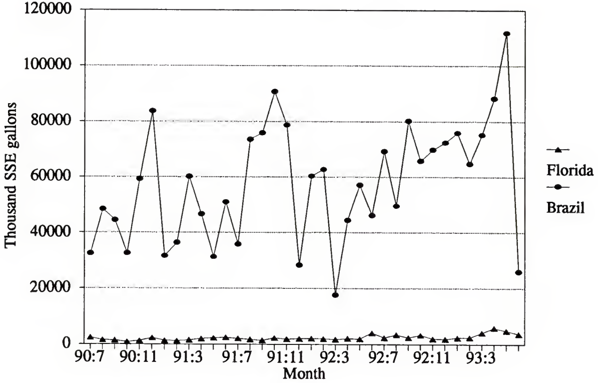

Figure D-4
FCOJ movement in the European market

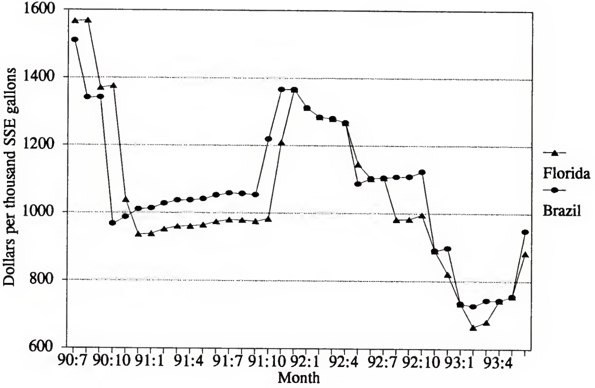

Figure D-5
FCOJ price series (deflated) in the US

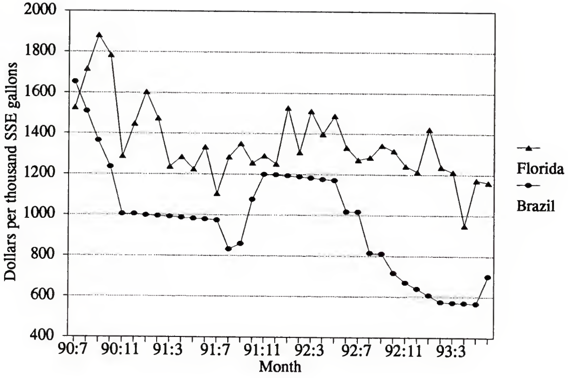

Figure D-6
FCOJ price series (deflated) in Europe

Estimation of Monthly Brazilian FCOJ Production

Since monthly Brazilian FCOJ production data for 1990/91 to 1992/93 seasons were not available (only annual production data were available), the values were estimated based on the available previous monthly production (1983/84 - 1989/90, Figure D-7). Figure D-7 suggests a monthly pattern. The estimation is based on a simple procedure. The ratio of the production level in each month to the total production in the corresponding year was calculated for that period (1983/84 - 1989/90). Then the average ratio for every month was used to estimate the monthly production in (1990/91 - 1992/93) seasons, based on the available annual production data for that period.

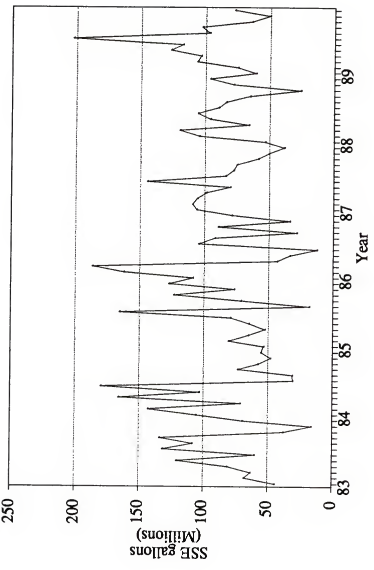

Figure D-7
Brazilian FCOJ monthly production

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He was married to Ratih Prabandari in 1987, and both were blessed with a baby boy, Juang Nurramadhan Taufik, in March 1991.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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